The SOLUTIONS FOR PART IV of the HOMEWORK #7 ASSIGNMENT FOR FALL 2024 m408 D

Sec 11.11 #14 (a)

$f(x) = x^{-\frac{1}{2}}$, $a = 4$, $n = 2$, Interval $c \le x \le d$ is $3.5 \le x \le 4.5$.

Determine the $n=2$ Taylor Polynomial for $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ centered at $a=4$.

Soln.:

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$	$\frac{1}{2}$
1	$-\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2}\left(\frac{1}{(\sqrt{x})^3}\right) = -\frac{1}{2x\sqrt{x}}$	$-\frac{1}{16}$
2	$\frac{3}{4}x^{-\frac{5}{2}} = \frac{3}{4}\left(\frac{1}{(\sqrt{x})^5}\right) = \frac{3}{4x^2\sqrt{x}}$	$\frac{3}{128}$

$$T_2(x) = \frac{\frac{1}{2}}{0!} - \frac{\frac{1}{16}}{1!}(x-4) + \frac{\frac{3}{128}}{2!}(x-4)^2$$

$$\boxed{T_2(x) = \frac{1}{2} - \frac{1}{16}(x-4) + \frac{3}{256}(x-4)^2}$$

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Sec 11.11, #14(a) (continued):

Recall that $f(x) = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$ and

$$T_2(x) = \frac{1}{2} - \frac{1}{16}(x-4) + \frac{3}{256}(x-4)^2 \text{ and}$$

The Interval $c \leq x \leq d$ is $3.5 \leq x \leq 4.5$.

At $x = c = 3.5$,

$$T_2(3.5) = \frac{1}{2} - \frac{1}{16}(3.5-4.0) + \frac{3}{256}(3.5-4.0)^2$$

$$T_2(3.5) = \frac{1}{2} - \frac{1}{16}(-0.5) + \frac{3}{256}(-0.5)^2.$$

Since $0.5 = \frac{1}{2}$,

$$T_2(3.5) = \frac{1}{2} + \frac{1}{16} \cdot \frac{1}{2} + \frac{3}{256} \cdot \frac{1}{4}$$

$$T_2(3.5) = \frac{1}{2} + \frac{1}{32} + \frac{3}{1024} = \frac{512}{1024} + \frac{32}{1024} + \frac{3}{1024}$$

$$T_2(3.5) = \frac{547}{1024} = 0.5341797$$

$$f(3.5) = \frac{1}{\sqrt{3.5}} = 0.5345225$$

$$|f(3.5) - T_2(3.5)| = |0.5345225 - 0.5341797|$$

$$|f(3.5) - T_2(3.5)| = 0.0003428$$

= THE ERROR IN USING $T_2(3.5)$ to Approximate $f(3.5)$,

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Sec 11.11, #14(a) (Continued).

At $x = d = 4.5$,

$$T_2(4.5) = \frac{1}{2} - \frac{1}{16}(4.5-4.0) + \frac{3}{256}(4.5-4.0)^2$$

$$T_2(4.5) = \frac{1}{2} - \frac{1}{16}(0.5) + \frac{3}{256}(0.5)^2.$$

Since $0.5 = \frac{1}{2}$,

$$T_2(4.5) = \frac{1}{2} - \frac{1}{16} \times \frac{1}{2} + \frac{3}{256} \times \frac{1}{4}$$

$$T_2(4.5) = \frac{1}{2} - \frac{1}{32} + \frac{3}{1024} = \frac{512}{1024} - \frac{32}{1024} + \frac{3}{1024}$$

$$T_2(4.5) = \frac{483}{1024} = 0.4716797$$

$$f(4.5) = \frac{1}{\sqrt{4.5}} = 0.4714045$$

$$|f(4.5) - T_2(4.5)| = |0.4714045 - 0.4716797|$$

$$|f(4.5) - T_2(4.5)| = |-0.0002752|$$

$$|f(4.5) - T_2(4.5)| = 0.0002752$$

= The ERROR IN USING $T_2(4.5)$ to Approximate $f(4.5)$.

Sec 11.11

#16(a) $f(x) = \sin x$, $a = \pi/6$, $n = 4$.

Determine the $n=4$ Taylor Polynomial, $T_4(x)$.

Soln:

n	$f^{(n)}(x)$	$f^{(n)}(\pi/6)$
0	$\sin x$	$\sin(\pi/6) = \frac{1}{2}$
1	$\cos x$	$\cos(\pi/6) = \frac{\sqrt{3}}{2}$
2	$-\sin x$	$-\sin(\pi/6) = -\frac{1}{2}$
3	$-\cos x$	$-\cos(\pi/6) = -\frac{\sqrt{3}}{2}$
4	$\sin x$	$\sin(\pi/6) = \frac{1}{2}$

$$T_4(x) = \frac{\frac{1}{2}}{0!} + \frac{\frac{\sqrt{3}}{2}}{1!}(x-\pi/6) + \frac{-\frac{1}{2}}{2!}(x-\pi/6)^2 + \frac{-\frac{\sqrt{3}}{2}}{3!}(x-\pi/6)^3 + \frac{\frac{1}{2}}{4!}(x-\pi/6)^4$$

$$\boxed{T_4(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x-\pi/6) - \frac{1}{4}(x-\pi/6)^2 - \frac{\sqrt{3}}{12}(x-\pi/6)^3 + \frac{1}{48}(x-\pi/6)^4}$$

NOTE: $3! = 6$ and $4! = 24$.

Sec 11.11 # 19 (a).

$f(x) = e^{x^2}$, $a=0$, $n=3$,

The interval $c \le x \le d$ is $0 \le x \le 0.1$.

Determing the $n=3$ Taylor Polynomial for $f(x) = e^{x^2}$ centered at $a=0$:

Sol'n :

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	e^{x^2}	1
1	$2x e^{x^2}$	$0 \times 1 = 0$
2	$2e^{x^2} + 2x(2xe^{x^2})$ $= 2e^{x^2} + 4x^2 e^{x^2}$ $= e^{x^2}(2+4x^2)$	$1(2+0) = 2$
3	$(2xe^{x^2})(2+4x^2) + e^{x^2}(8x)$ $= e^{x^2}(4x+8x^3) + 8xe^{x^2}$ $= e^{x^2}(12x+8x^3)$	$1(0+0) = 0$

$$T_3(x) = \frac{1}{0!} + \frac{0}{1!}(x-0) + \frac{2}{2!}(x-0)^2 + \frac{0}{3!}(x-0)^3$$

$$T_3(x) = 1 + 0x + 1x^2 + 0x^3$$

$$\boxed{T_3(x) = 1 + x^2}$$

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Sec 11.11 #19 (a) (Continued). Recall $c \le x \le d$ is $0 \le x \le 0.1$

At $x = c = 0$. Recall That $T_3(x) = 1 + x^2$.

$T_3(0) = 1 + 0^2 = 1$

$f(0) = e^{0^2} = e^0 = 1$

$|f(0) - T_3(0)| = |1 - 1| = 0$

= The ERROR in USING $T_3(0)$ to approximat $f(0)$.

At $x = d = 0.1$

$T_3(0.1) = 1 + (0.1)^2 = 1.0100000$

$f(0.1) = e^{0.01} = 1.0100502$

$|f(0.1) - T_3(0.1)| = |1.0100502 - 1.0100000|$

$= |0.0000502|$

$|f(0.1) - T_3(0.1)| = 0.0000502 =$

THE ERROR IN USING $T_3(0.1)$ To Approximate $f(0.1)$.