

**Midterm Spring 2019 #5(c)**

The matrix  $\begin{pmatrix} 3 & 13 \\ 20 & 19 \end{pmatrix}$  can be written as  $LDU$ , where  $L$  is a vertical shear,  $D$  is diagonal, and  $U$  is a horizontal shear.

**Quiz 1 #2(b)**

Suppose  $T$  is a linear transformation from  $\mathbb{R}^3$  to  $\mathbb{R}^2$  such that

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 5 \\ 14 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Find the matrix of  $T$ .

**Midterm Spring 2017 #4(a)**

Let  $C_k = (A - kI_4)(B - kI_4)$ , where

$$A = \begin{bmatrix} 1 & 2 & 6 & 1 \\ 0 & 3 & \sqrt{7} & 2 \\ 0 & 0 & 4 & 7 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 10 & 1 & 0 & 0 \\ 9 & 0 & 7 & 0 \\ 0 & \sqrt{2} & 1 & 6 \end{bmatrix}$$

Find all values of  $k$  such that  $C_k$  is *not* invertible.

**Midterm Spring 2017 #5(b)**

Given two distinct lines  $L_1$  and  $L_2$  in  $\mathbb{R}^2$  passing through the origin, there exists a  $2 \times 2$  matrix  $A$  such that  $\text{Im}(A) = L_1$  and  $\ker(A) = L_2$ . Explicitly write down one such matrix if  $L_1 : y = x$  and  $L_2 : y = 2x$ .

**Midterm Spring 2018 #3(d)**

Let  $V$  be the subspace of  $\mathbb{R}^4$  spanned by the vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

Let  $W$  be the subspace of  $\mathbb{R}^4$  spanned by the vectors

$$\vec{w}_1 = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{w}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

Find a basis for  $V \cap W$ .

**Midterm Spring 2019 #2(a)**

In  $\mathbb{R}^4$ , let  $V$  be the hyperplane  $x+2y-z-w=0$ , and let  $W$  be the hyperplane  $-x-y+z-3w=0$ . Find a basis of  $V \cap W$ . Extend it to a basis of  $V$ .