SMMG February 4th, 2006

featuring Dr. Lorenzo Sadun

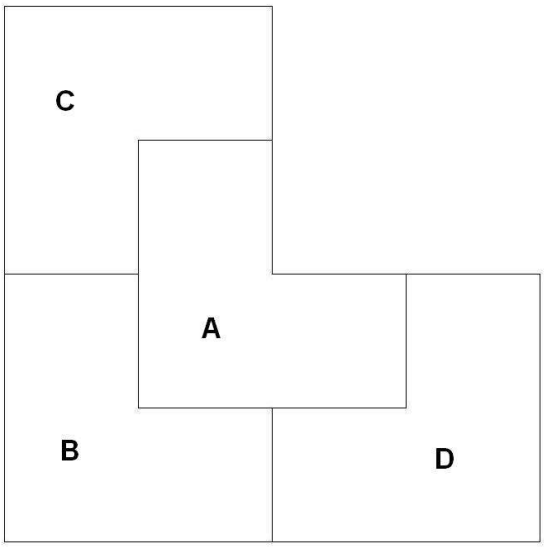
"Substitution Tilings and the Droste Effect"

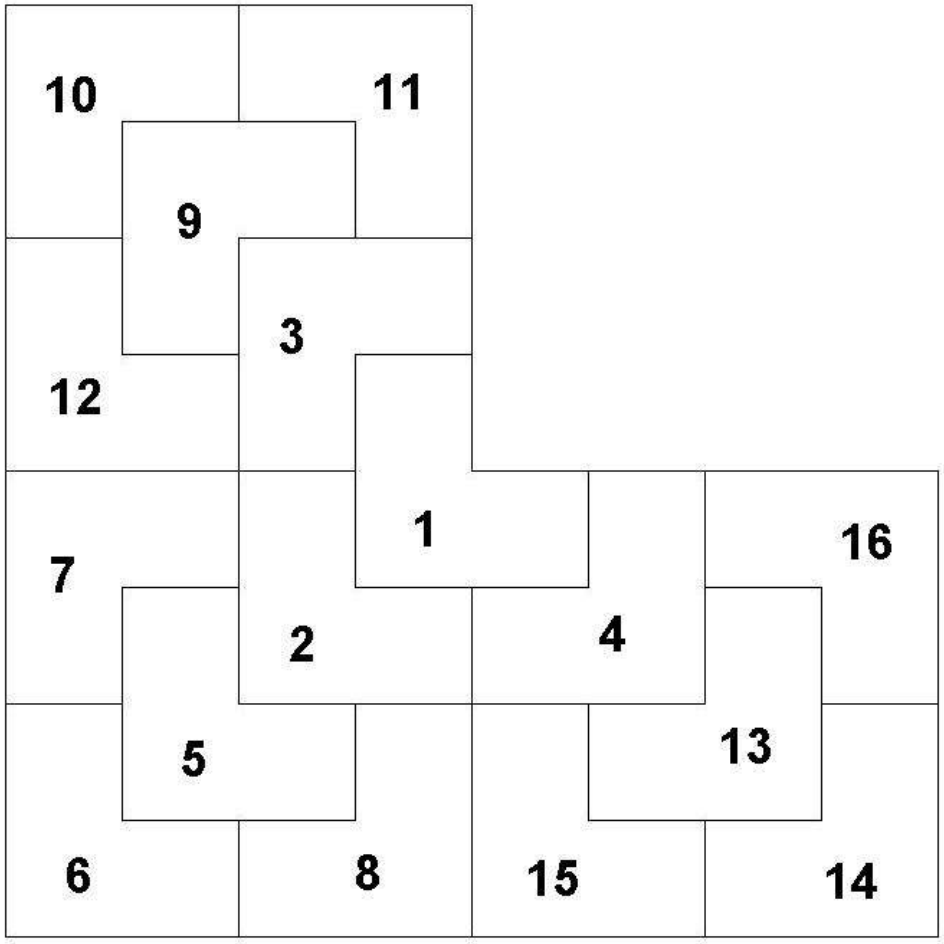
1. We gave you a bunch of paper squares. Cut out one corner as marked so that you get an L-shaped tile (also called a chair tile).

    a) First take four of these L-tiles and put them together to make a bigger L-shape.

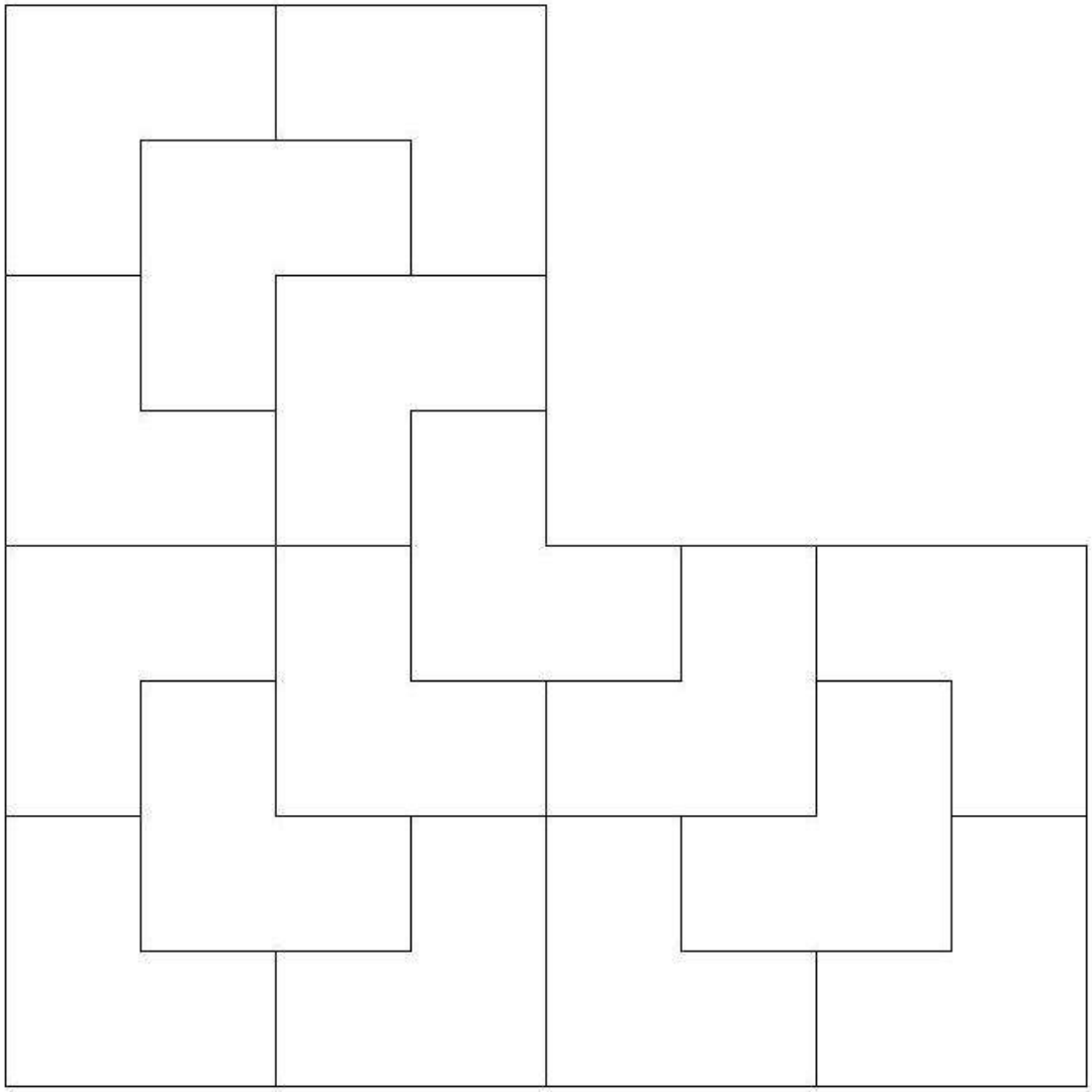
    b) Now take 16 of your prototiles and arrange them so they form one big L again. Is there only one way of doing it? Compare with your neighbor!

2. Here is a picture of the figures you created earlier of 4, respectively 16 prototiles.

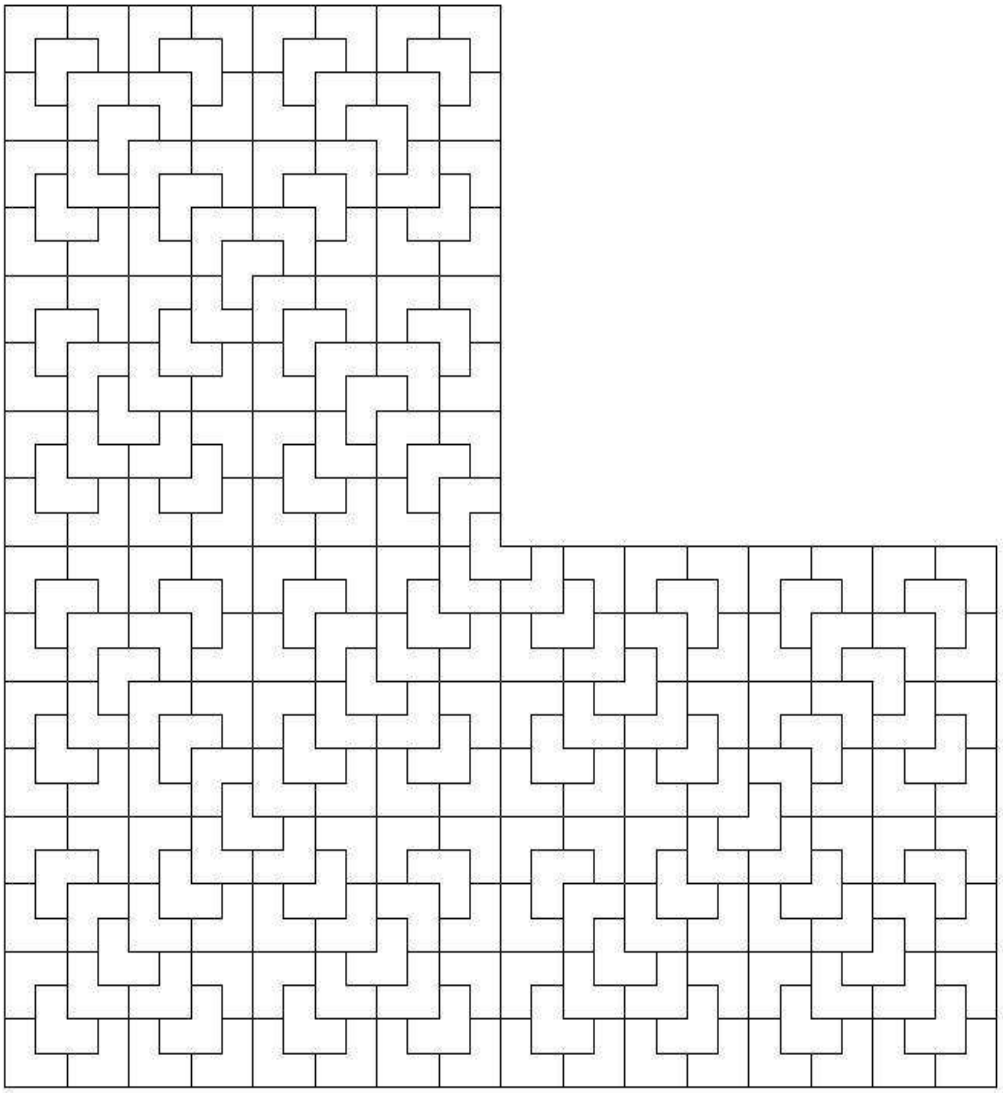




3. Notice that if you compare the position of tile #**1** with the rest of the tiles, some of them are just translated while others are rotated by 90, 180 or 270 degrees. Take four different colors to mark each of those in the following picture. How many of each type occur?

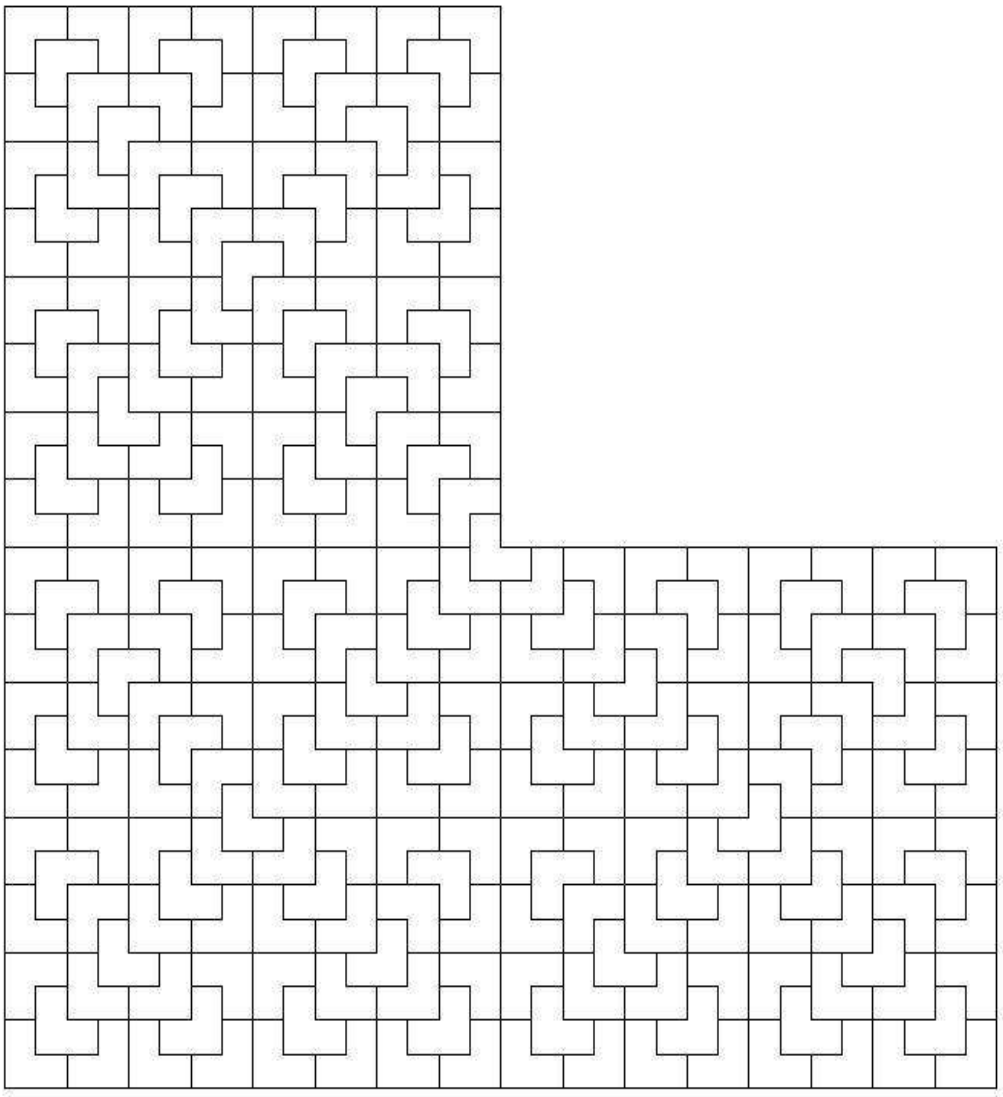


4. If you had cut out 1024 chair tiles, you could have made this picture. Take a colored pen and divide this picture up into four clusters of 256 tiles each.
   Next, with a different color, divide the center one (i.e. tile in the position of **A**) and divide it into four clusters of 64 tiles. Take tile in position **A** in there and, using a different color, divide it into four clusters of 16 tiles. Repeat with a forth color by dividing position **A** into four clusters of 4 tiles.



   Which tile do you end up with?
   Notice that the picture you have created looks like you have zoomed in on the picture! Mark the point that seems to be the center of the zooming.

5. Repeat the process of the previous problem, except that after breaking it up into four pieces of 256 you now pick tile in position **C** each time.

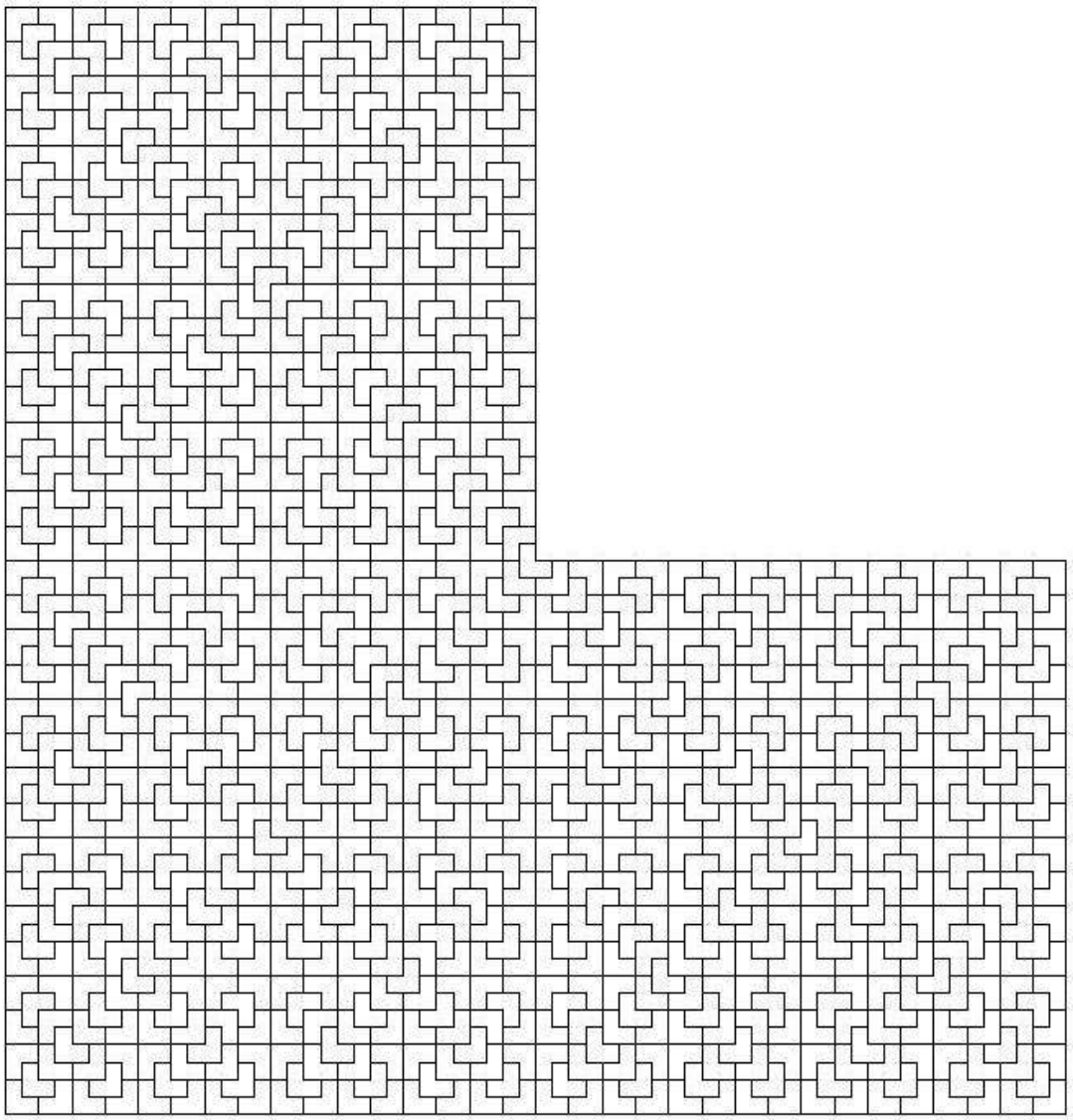


You should end up with a different tile and a different center point for your zooming process! Mark it!

What is the position of this one as opposed to the center tile in the previous activity?

6. We have divided each of the tiles one more time. How many tiles does this make?

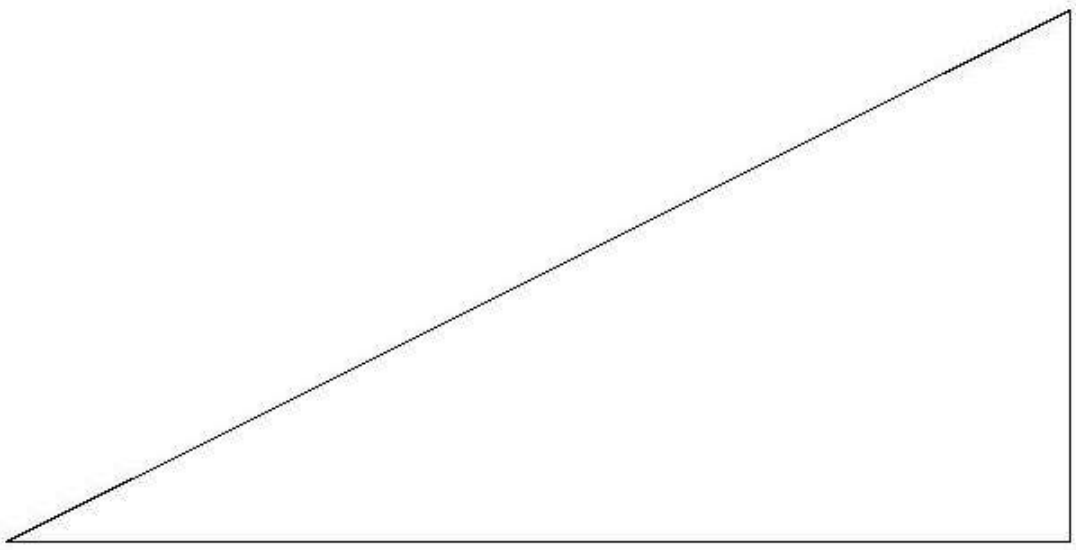
   Start by dividing this figure into 16 sub-chairs. Now keep breaking this up by picking the tile in position **5** each time.



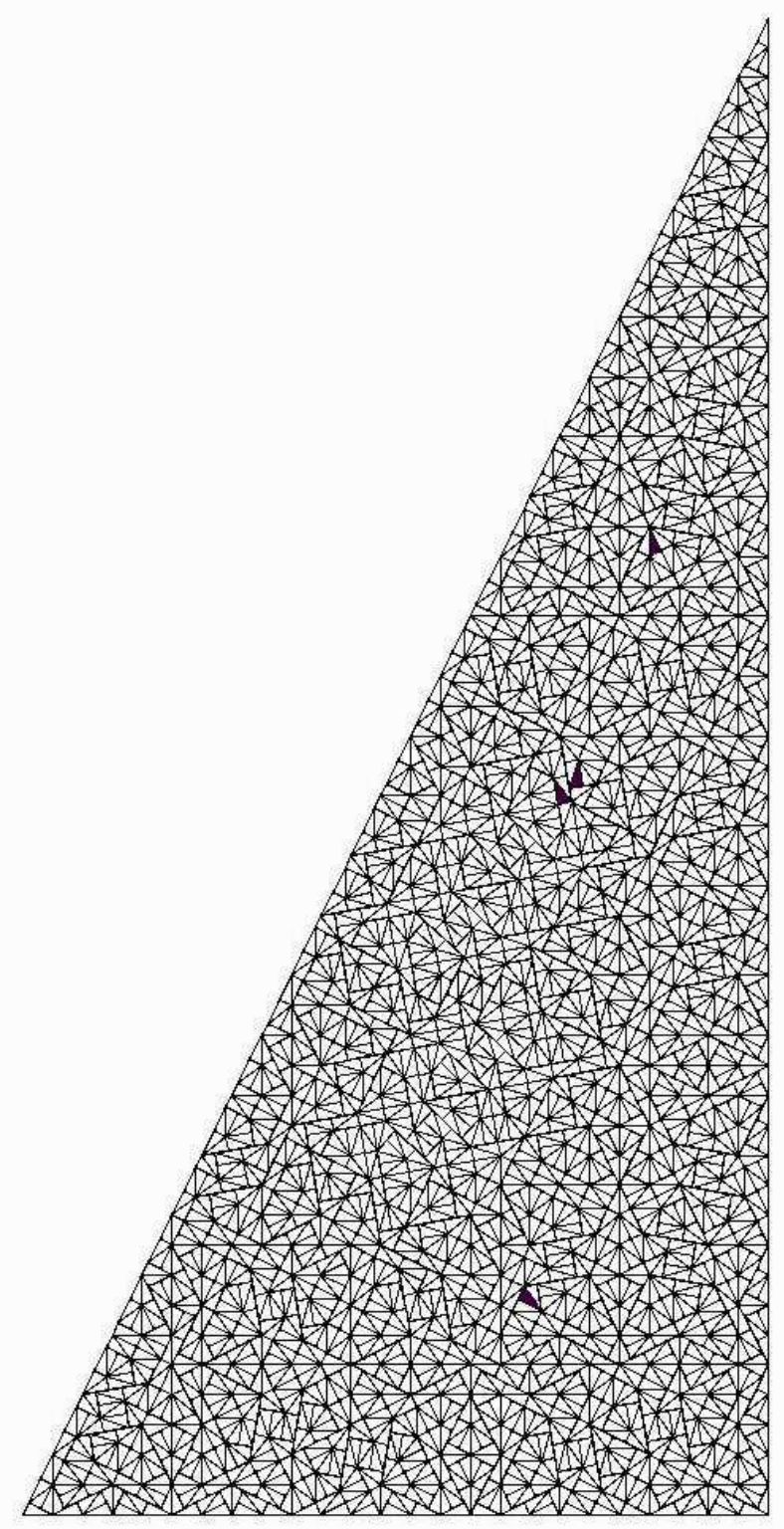
7. What happens when you repeat with tiles number **2** or **4**, respectively?

8. The next figure gives you a so-called pinwheel tile. It can be constructed by starting with a right angle triangle with side lengths of 1, 2 and square root of 5. We will reverse the process you first did with your paper chair tiles and ask you to subdivide this tile into smaller tiles of the same shape.
   Is there just one way of doing this?

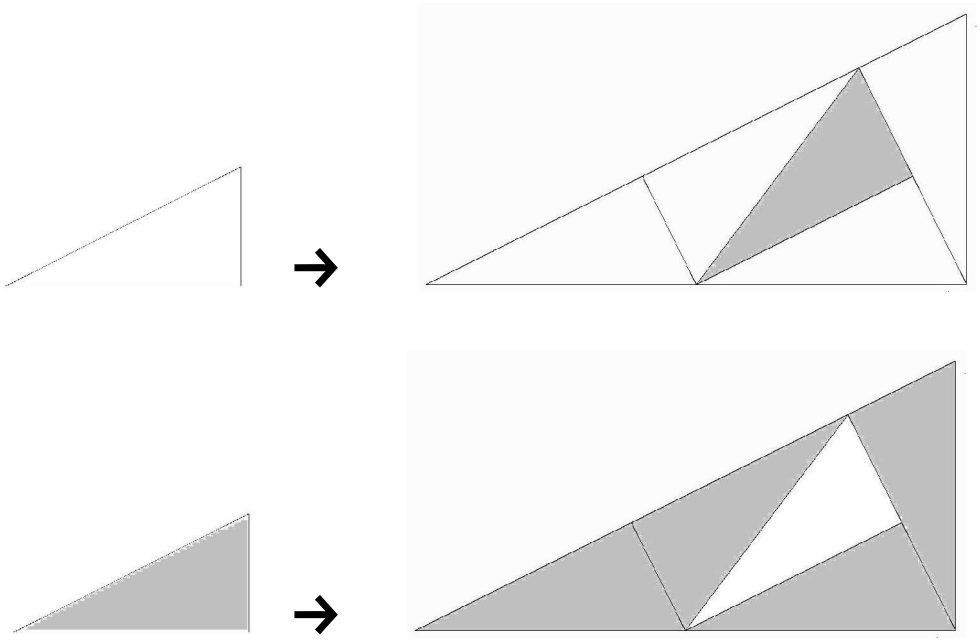
   Now suppose you only want subtiles with hypotenuse of length 1. How many subtiles did you create?



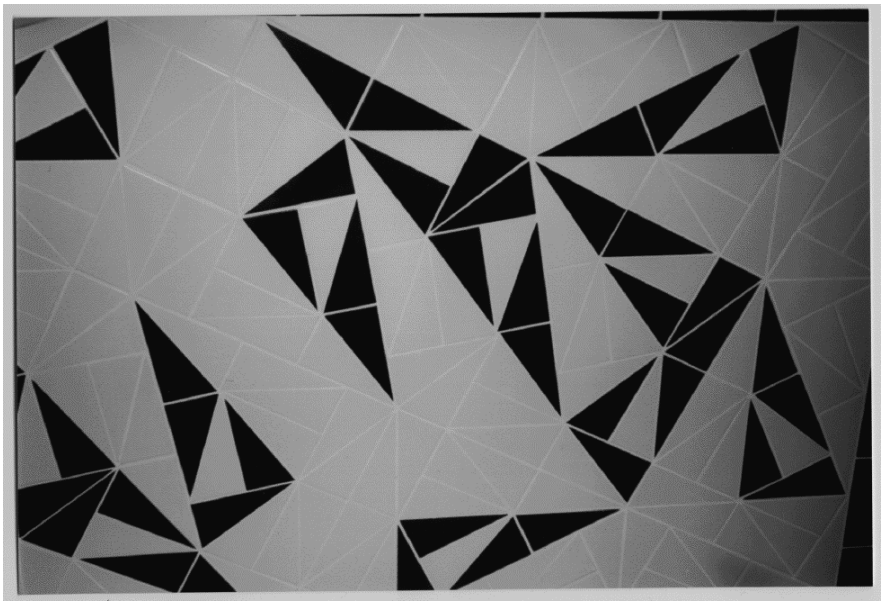
9. Attached is a picture of a pinwheel pattern. For each of the four tiles marked, draw the 5-unit super-tile and the 25-unit super-super-tile of which it is part.



10. Here is something to take home and try: Imagine you subdivide a pinwheel tile by the following substitution rule, using two different colors.



Take a piece of paper and draw what the resulting pattern looks like! Now you have something that looks just like the bathroom floor in Dr. Sadun's home ☺



11. Here is a pretty example of a Penrose tiling.
    Have fun examining it!