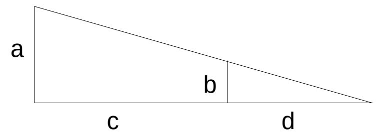
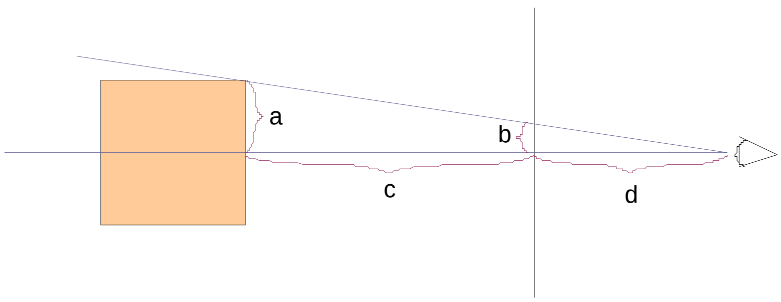
SMMG April 1st, 2006
featuring Zachary Booth Simpson
"Video Game Mathematics"
Mathematics of 3D Computer Animation

Perspectometer





SimilarTriangles:

a is to *c*+*d* as *b* is to *d*:

$$\frac{a}{c+d}=\frac{b}{d}$$

$$a=\frac{b}{d}(c+d)$$

- Hold a ruler up, close one eye and measure the apparent height of the line on the board in inches.

- Have your friend measure the length of your arm (from your eye to the ruler) in inches.

- Estimate the distance to the board:
  Each row of seats is _____ feet.
  From the board to the first row of seats it is _____ feet.

- Estimate the size of the line by similar triangles.

- What are the units of length for your estimate?

Dot Product

If $\vec{a}=\begin{bmatrix} a_x \\ a_y \end{bmatrix}$ and $\vec{b}=\begin{bmatrix} b_x \\ b_y \end{bmatrix}$, then the dot product is:

$\vec{a}\cdot\vec{b}=\|\vec{a}\|\cdot\|\vec{b}\|\cdot\cos\,\theta$ $\qquad = a_x\cdot b_x + a_y\cdot b_y$

$\vec{a}=\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\quad \left(\text{ so } \|\vec{a}\|=\sqrt{(-2)^2+1^2}=\sqrt{5}=2.236\ldots.\right)$

$\vec{b}=\begin{bmatrix} 7 \\ 24 \end{bmatrix}$ $\quad \left(\text{ so } \|\vec{b}\|=\sqrt{7^2+24^2}=\sqrt{625}=25\right)$

$\vec{a}\cdot\vec{b}=$

Are $\vec{a}$ and $\vec{b}$ orthogonal?

Length of projection of $\vec{a}$ onto $\vec{b}$: $\dfrac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}=$

What vector would be in the same direction as $\vec{b}$, but have unit length?

Is the face with normal vector $\vec{a}$ visible if the camera vector is $\vec{b}$?

$\vec{a}=\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\quad \left(\text{ so } \|\vec{a}\|=\sqrt{3^2+4^2}=\sqrt{25}=5\right)$

$\vec{b}=\begin{bmatrix} 8 \\ -6 \end{bmatrix}$ $\quad \left(\text{ so } \|\vec{b}\|=\sqrt{8^2+(-6)^2}=\sqrt{100}=10\right)$

$\vec{a}\cdot\vec{b}=$

Are $\vec{a}$ and $\vec{b}$ orthogonal?

Length of projection of $\vec{a}$ onto $\vec{b}$: $\dfrac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}=$

What vector would be in the same direction as $\vec{b}$, but have unit length?

Is the face with normal vector $\vec{a}$ visible if the camera vector is $\vec{b}$?

Multiplying Matrices with Vectors & Other Matrices

Examples:

$$\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 7 \end{bmatrix} = 4\begin{bmatrix} 3 \\ 4 \end{bmatrix} + 7\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \end{bmatrix} + \begin{bmatrix} -7 \\ 14 \end{bmatrix} = \begin{bmatrix} 5 \\ 30 \end{bmatrix}$$

For more than one column, just repeat for each column.

$$\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -5 & -9 \\ 5 & 8 & -1 \end{bmatrix}$$

Multiply the following:

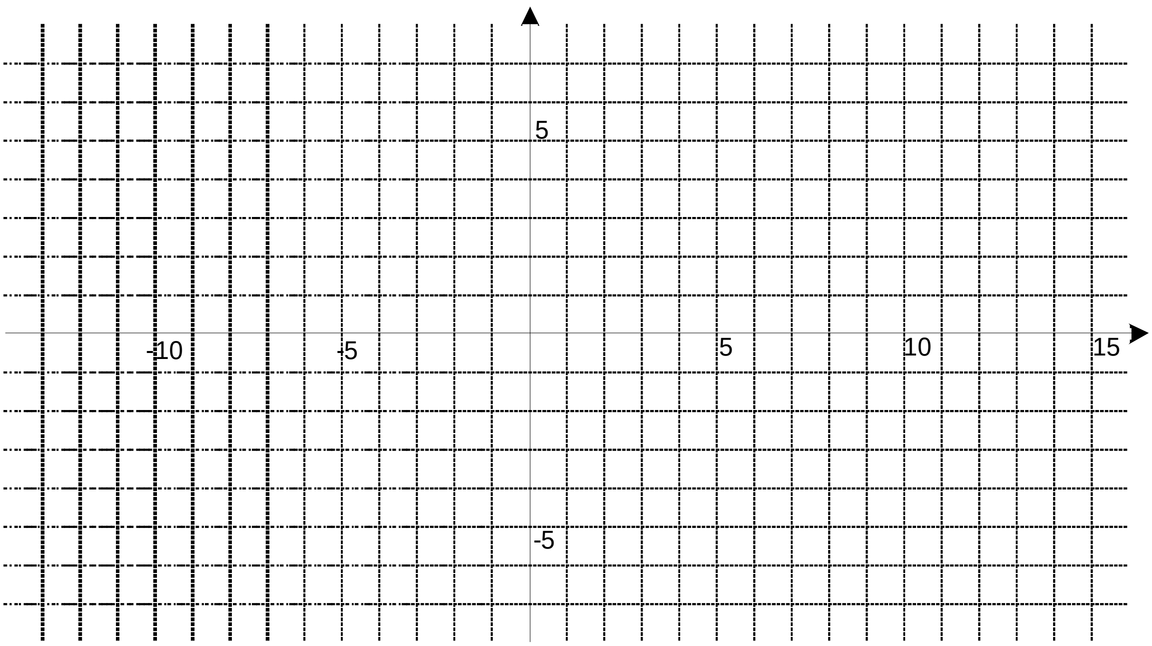
$$\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & -1 & -5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 0 & -1 \\ 0 & 0 & -3 & -2 \\ 1 & 1 & 1 & 1 \end{bmatrix} =$$

Matrices & Transformations

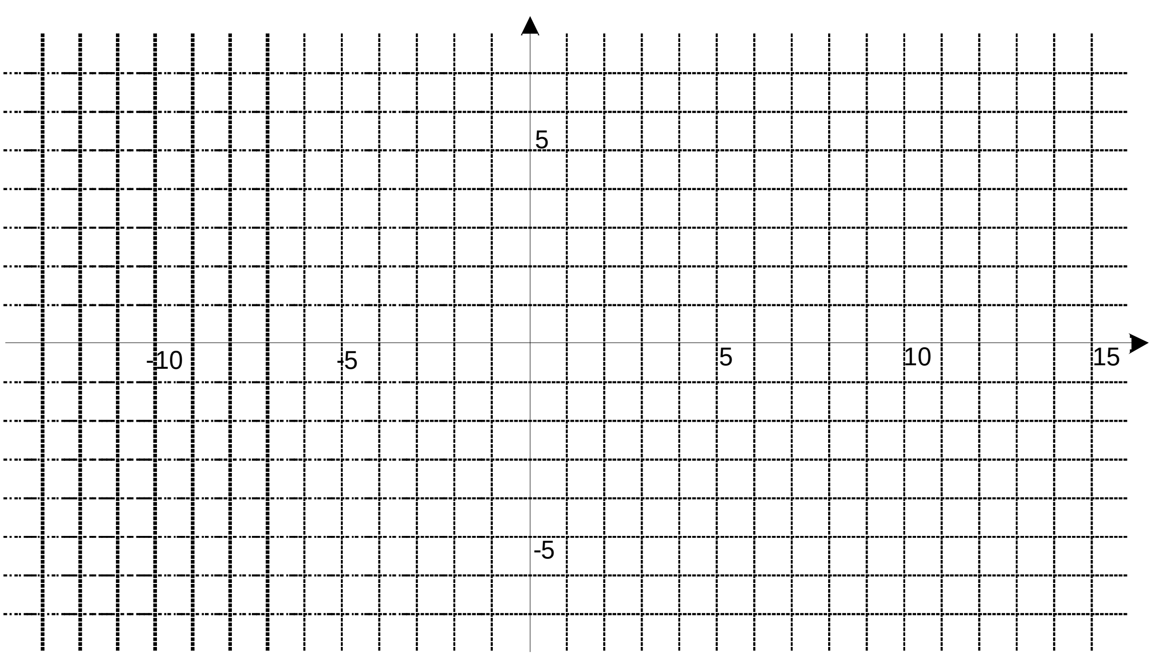
Plot the following vertices, and join the dots in order to form a figure.

$$\begin{bmatrix}1\\0\end{bmatrix} \quad \begin{bmatrix}1\\1\end{bmatrix} \quad \begin{bmatrix}0\\1\end{bmatrix} \quad \begin{bmatrix}2\\3\end{bmatrix}$$

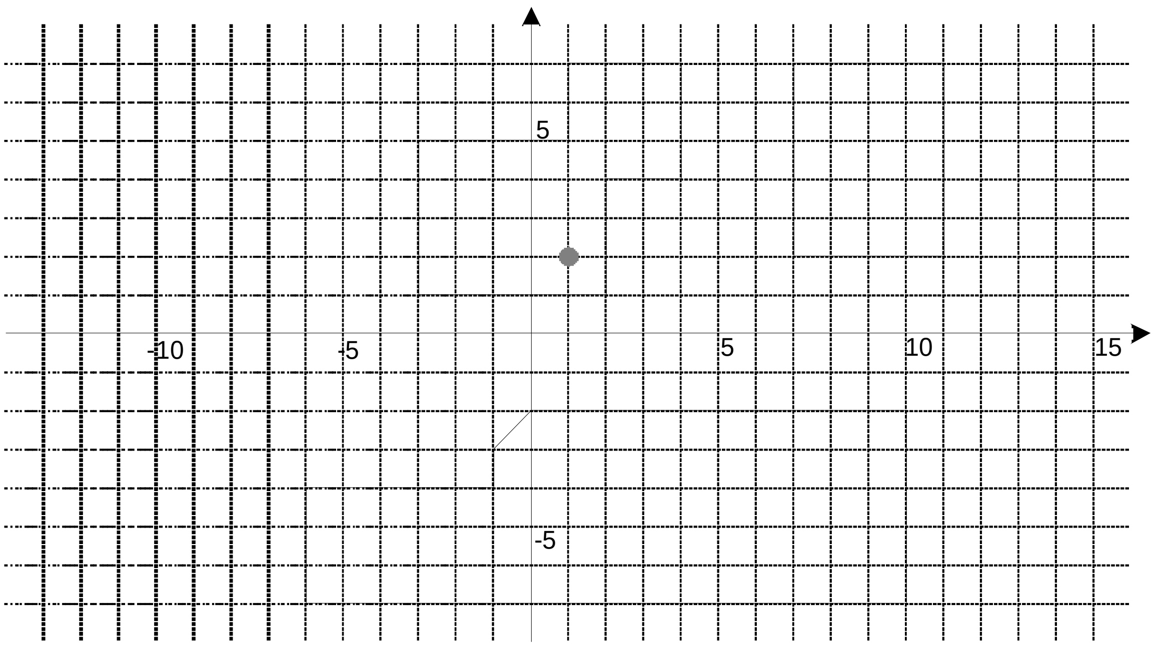


Apply the following transformations by multiplying the matrix with each vector (or doing it all at once). Draw the resulting figures. Don't forget to do add the extra 1 at the end of each vector to make it a 3-entry vector! ***Think before you compute.*** If you visualize the transformation instead of blindly computing it will be a lot easier!

$$\begin{bmatrix}1 & 0 & 6\\0 & 1 & 3\\0 & 0 & 1\end{bmatrix}, \begin{bmatrix}1 & -2 & 0\\1 & 2 & 0\\0 & 0 & 1\end{bmatrix}, \begin{bmatrix}1 & -2 & 5\\1 & 2 & -1\\0 & 0 & 1\end{bmatrix}$$



Matrix Minesweeper



$$\begin{bmatrix} +3 & +1 \\ +2 & +2 \end{bmatrix} \quad \begin{bmatrix} -2 & +1 \\ +1 & +1 \end{bmatrix} \quad \begin{bmatrix} -3 & +2 \\ 0 & -1 \end{bmatrix}$$