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"Pigeonholes, Shaking Hands, and Kevin Bacon: An Introduction to Ramsey Theory"

The Pigeonhole Principle

The Pigeonhole Principle states that if you put m pigeons into n holes, and if m>n, then at least one hole must have more than one pigeon in it.

For example, in the image on the left, there are 9 cubby holes and 7 pigeons. What if you wanted to add three more pigeons? Since only two holes are left, two of the pigeons would have to share a hole.

See if you can figure out how the following problems relate to the pigeonhole principle:

1. How do you know that at least two people in New York have exactly the same number of hairs on their head?
2. There are 81 people in a room, some of whom have shaken hands with other people in the room. How do you know that at least two people shook the same number of hands?

Ramsey Theory

Ramsey Theory, named for Frank P. Ramsey, is a branch of mathematics that studies the conditions under which order must appear. Problems in Ramsey theory typically ask a question of the form: how many elements must there be in a set to guarantee that a particular property will hold? The Pigeonhole Principle could be stated in a different way: How many holes do you need to know that if you have *n* pigeons, two of them have to share a hole? This is a basic Ramsey Theory example.

Another example is the following:

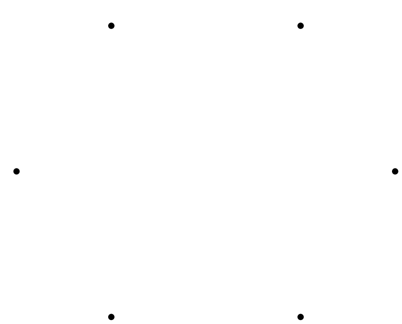
The Ramsey Party Problem: Suppose you have a room full of people (like at a party). Some of them have shaken hands and some of them haven't. Let *n* be any number (like 3, 12, or 104). How many people should be in the room so that you can be sure that either exactly *n* of them have shaken each other's hands or exactly *n* of them have not shaken each other's hands (that is, every person in *that group* didn't shake hands with anybody else in *that group*)?

The numbers that you get are the so-called *Ramsey Numbers*, and are denoted by $R(n)$. For the most part, these numbers are really hard to find, although some bounds for them are known. Here is a table for some Ramsey Numbers:

n	*R(n)*
2	2
3	6
4	18
5	43-49
6	102-165

Mathematicians think about the party problem in terms of graphs. Suppose dots represent people and you draw a red line between two dots if those two people did not shake hands and a blue line if they did. Then, for example, finding $R(3)$ is equivalent to answering the following question: How many dots (or vertices) do you need so that a complete graph (this means that all the dots are connected to each other) in two colors (this means that we are coloring each edge in one of two colors, red or blue) *always* has a red triangle or a blue triangle? From the table above you know that the answer is 6 vertices.

Given the dots below, can you draw a graph as explained in the above paragraph? (Remember, you need two colors, and a triangle in one of the two colors.)

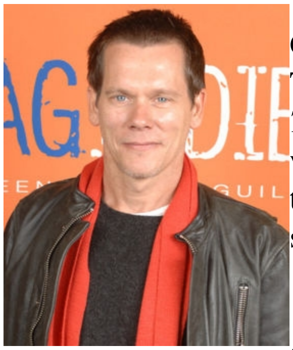


There is a more general theorem that extends the results above:

The Infinite Ramsey Theorem: Given an infinite number of people at a party, there will always be an infinite subset of people that shook hands with everyone else in that subset, or an infinite number of people who didn't.

There are a lot more theorems that are important to this theory, but the ones mentioned in this packet are the most basic and fundamental.

Six Degrees of Kevin Bacon (taken from Wikipedia)



The game was invented in 1994 by Craig Fass, Brian Turtle and Mike Ginelli while they were students at Albright College in Reading, Pennsylvania. They were watching television when a commercial for Bacon's most recent film *The Air Up There* came on and led to a discussion about his film career and the wide variety of actors he had thus so far worked with. According to Turtle, the title of the game was suggested by a producer on *The Jon Stewart Show* when the students made an appearance demonstrating the concept.

The concept is simple, but finding the smallest number of links can be difficult. The way you link an actor with Bacon is like so:

1. Pick any film actor in history.

2. Link the actor you've chosen to Bacon via the films they've shared with other actors until you end up with Kevin Bacon himself.

Here is an example, using Elvis Presley:

1. Elvis Presley was in *Change of Habit* (1969) with Edward Asner.

2. Edward Asner was in *JFK* (1991) with Kevin Bacon.

Therefore Elvis Presley has a Bacon number of at most 2. (The only way to show that it is exactly 2 is to show that there isn't any movie that they both acted in.)

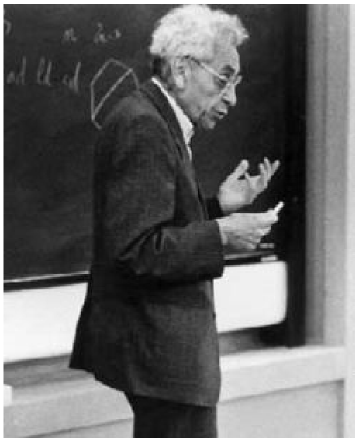
The game has expanded to become more about linking two film actors than just one actor to Kevin Bacon. The name is still retained, however. Also, even thought Kevin Bacon is very well-connected, the person who can be connected to any other actor in the least average number of steps is Dennis Hopper.

For a website that links Bacon to anybody else, go to http://oracleofbacon.org/. To link any two actors together, go to http://oracleofbacon.org/star_links.html.

About Frank P. Ramsey

Frank Plumpton Ramsey (February 22, 1903 – January 19, 1930) was a British mathematician who, in addition to mathematics, made significant contributions in philosophy and economics. One of the theorems proved in his 1930 paper *On a problem of formal logic* sparked a whole field of study we now know as *Ramsey Theory.* Unfortunately, what looked like the beginning of a brilliant career ended with his tragic death at the age of 26.

About Paul Erdos



Paul Erdős, also Pál Erdős, in English Paul Erdos or Paul Erdös (March 26, 1913 – September 20, 1996), was an immensely prolific (and famously eccentric) Hungarian mathematician who, with hundreds of collaborators, worked on problems in combinatorics, graph theory, number theory, classical analysis, approximation theory, set theory and probability theory. He was the main developer of Ramsey Theory. There is a famous quote by him about the difficulty of computing Ramsey Numbers:

"Imagine an alien force, vastly more powerful than us landing on Earth and demanding the value of R(5) or they will destroy our planet. In that case, we should marshal all our computers and all our mathematicians and attempt to find the value. But suppose, instead, that they asked for R*(6), we should attempt to destroy the aliens".*

There are a couple of really great books about Erdős' life, *The Man Who Loved Only Numbers,* by Paul Hoffman, and *My Brain is Open,* by Bruce Schechter. There is also a very interesting documentary about his life called *N is a Number: A Portrait of Paul Erdős.*