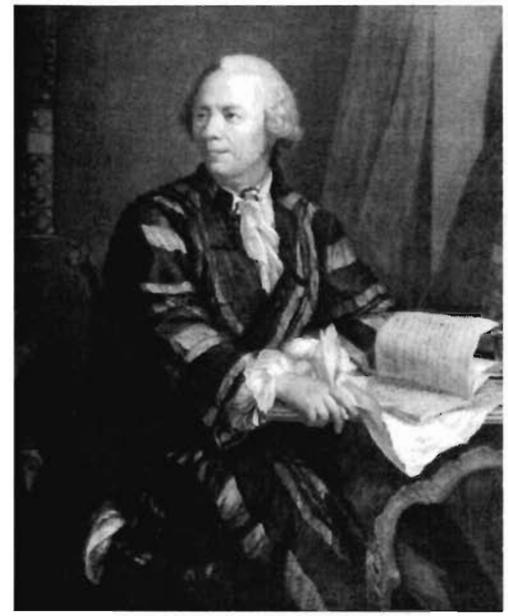
Images of Euler



Saturday Morning Math Group—Euler
Michael P. Saclolo

April 28, 2007

ACTIVITY I

In the exercise packet are several copies of portraits of Leonhard Euler. In the space provided below draw your own rendition of him and share it with the group.

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ACTIVITY II

Below is Euler's answer to the "Basel Problem", i.e. the sum of the series:

$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{k^2}+\ldots=$ is $\frac{\pi^2}{6}$. Based on this, can you figure out the sum of the following series?

1. $\frac{6}{1^2}+\frac{6}{2^2}+\frac{6}{3^2}+\ldots+\frac{6}{k^2}+\ldots=$

2. $\frac{3}{1^2}+\frac{3}{2^2}+\frac{3}{3^2}+\ldots+\frac{3}{k^2}+\ldots=$

3. $\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots+\frac{1}{(2k)^2}+\ldots=$,where k=1,2,3,.... This one is somewhat tricky. Hint: Is there a common factor?

4. $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\ldots+\frac{1}{(2k+1)^2}+\ldots=$,where k=0,1,2,.... This one is bit trickier! Hint: consider what was "left out" and what you got for #3.

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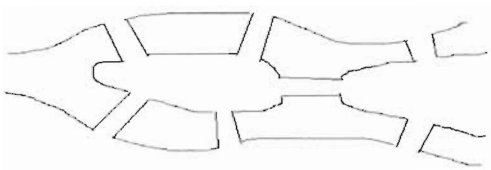
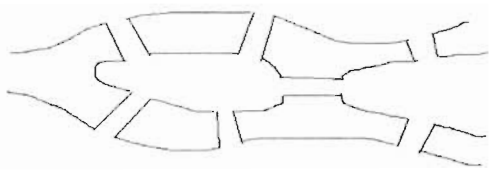
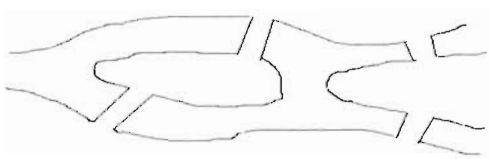
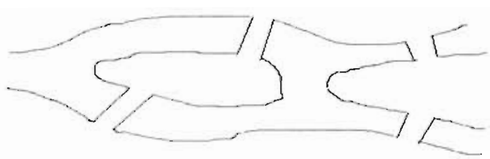
ACTIVITY III

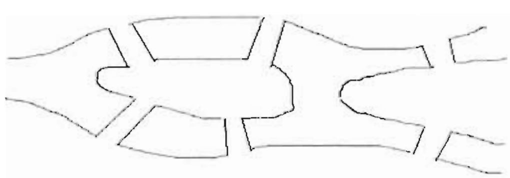
Below are several possibilities for the city of Königsberg with different numbers and positions of bridges. Can you draw a path that uses a bridge only once? Can you draw a path that uses a bridge only once while beginning and ending on the same land mass?

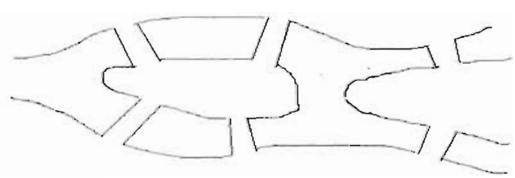


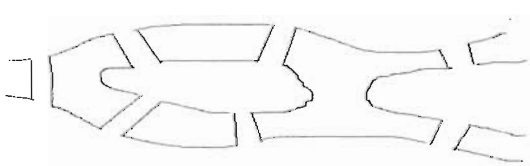


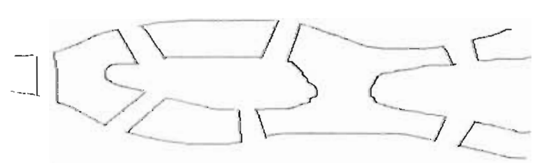












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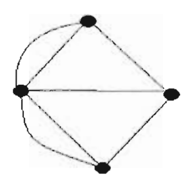
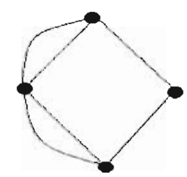
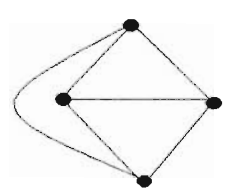
ACTIVITY VI

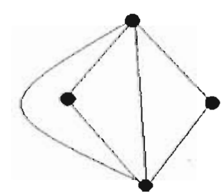
Nowadays we can use "graph theory" to answer the previous questions. We use "graphs" consisting of dots (called vertices (singular: vertex) or nodes) to represent land masses or towns and lines connecting dots (called edges) to represent bridges or roads. The number of edges touching a vertex is called the **degree** of that vertex. To have an **Euler circuit**, a path that begins and ends at the same vertex (land mass or town) using an edge only once then each vertex must have even degree. To have an open **Euler trail**, a path that uses an edge (road or bridge) exactly once but does not begin and end at the same vertex, then the graph must have exactly two vertices of odd degree.

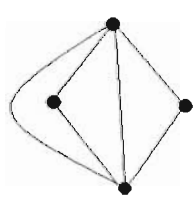
Below are several "graphs". Determine whether it is possible to have an open Euler trail and an Euler circuit.

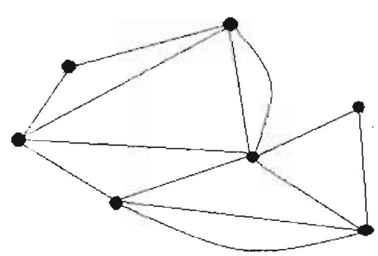












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ACTIVITY V

Below are a couple chessboards with one of the squares numbered. Using that square as a starting point, determine if there is a knight's tour possible. The easiest way is to use the numbers 2 to 64 to indicate where the next landing point of the knight would be.

							1

			1				

				1			

	1						

ACTIVITY VI

Take one of the geometric solids (a die or some other object). Count the number of vertices, faces, and edges and determine its Euler characteristic (V+F-E).

	V	F	E	V+F-E
Tetrahedron				
Hexahedron (Cube)				
Octahedron				
Dodecahedron				
Icosahedron				
Triangular prism				
Pyramid with square base				

ACTIVITY VII

The bulk of Euler's works were written in Latin. Many have not been translated into English. Try translating some of these titles of Euler's works. Use your intuition. See if some of the words resemble English words—particularly math terms (or French, Italian, or Spanish words that you know).

1. De summis serierum reciprocarum

2. Solutio problematis ad geometriam situs pertinentis

3. De fractionibus continuis dissertatio

4. De linea brevissima in superficie quaunque duo quaelibet puncta iungente

5. De solutione problematum Diophanterum per numeros integros

6. Methodus facilis computando angulorum sinus ac tangentes tam naturales quam artificiales

Hints: "De" can often be translated as "on", "about", as well as "from" or "of". You can use the word "essay" for "dissertatio". "Quaunque" can be translated as "that". "Diophanterum" = "of Diophantus". "Tam…quam" = "both … and" or "…as well as…".

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ACTIVITY VIII

Euler showed that prime numbers p of the form $p = 4n + 1$, where n is a positive integer can be written as a sum of two squares in a unique way, that is $p = a^2 + b^2$ for some positive integers a and b. See if you can figure out n as well as a and b for each of the following prime numbers.

Example: For p = 5, n = 1 while a = 1 and b=2 (since $5 = 1^2 + 2^2$).

Prime	n	a	b
13			
17			
29			
53			
117			