

# THE HAIRY BALL THEOREM

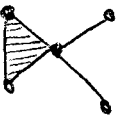
MATH CIRCLE 2-15-2008

## EXPLORATION PART I

### Part A

(1) Determine which of the following are simplicial complexes (make sure you know why).

a.)



b.)



c.)



d.)



(2) Find two simplicial decompositions for  $S^2$  (both different from the presented example).

(3) For each of your simplicial decompositions, compute the sum  $\chi = v - e + f$ , where  $v$ ,  $e$ , and  $f$  are the number of vertices, edges and faces that appear in your simplicial decomposition. What is  $\chi$  for the example I presented?

(4) Do you notice anything about these three values of  $\chi$ ?

(5) Can you make a conjecture about the value of  $\chi$  for different simplicial decompositions of  $S^2$ ?

\*(6) Can you prove your conjecture?

\* this is an advanced challenge