Part B

Investigation: On which closed 2-manifolds and for which positive integers, n, does there exist a smooth vector field with exactly **n non-trivial** zeros?

Exploration: S^2

(1) Can you find a smooth vector field on S^2 with exactly **two** zeros?

a. What is the index of each of these zeros?

b. What is the sum of the indices of all the zeros of this vector field?

(2) Can you find a vector field on S^2 with exactly **one** zero?

a. What is the index of this zero?

b. What is the sum of the indices of all the zeros of this vector field? Is this number familiar?

(3) If you had a vector field on S^2 with exactly **three** zeros, what might you guess the sum of the indices of these zeros would be?

(4) If you had a vector field on S^2 with exactly **n** zeros, what might you guess the sum of the indices of these zeros would be?

(5) Suppose the sum of the indices was independent of the choice of the vector field on S^2.

a. What would the sum be?

b. Is this value consistent with the existence of a smooth vector field on S^2 with no zeros? Why or why not?

c. How does this relate to the Hairy ball theorem?

Exploration: T^2

(6) Can you find a vector field on T^2 with **no** zeros?

a. What does this tell you about the sum of the indices?

(7) Can you find a vector field on T^2 with exactly **two** non-trivial zeros?

a. What is the index of each of these zeros?

b. What is the sum of the indices of these two zeros? Is this number familiar?

(8) Do you think there is a vector field on T^2 with exactly **one** non-trivial zero? Why or why not?

(9) If the sum of the indices was independent of the choice of the vector field on T^2, what do you think the sum would be?

Generalization

(10) If you had to guess, do you think the sum of the indices is independent of the choice of vector field for any closed 2-manifold?

a. If so, do you think this sum may be related to the genus of the 2-manifold? How?