Flip and Turn into Group Theory

Worksheet

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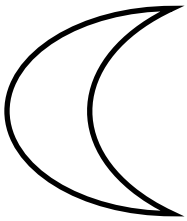
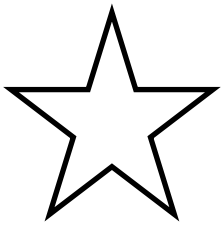
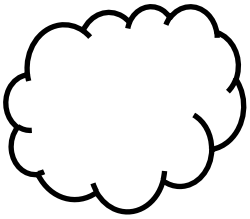
Finding symmetry in shapes

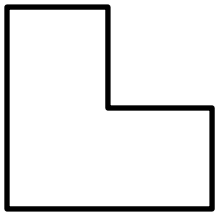
There are two types of symmetry that we want to look for in a shape:
An axis of symmetry is a line we can draw in a shape so that either side of the axis looks like the mirror image of the other side.
A rotational symmetry is when after rotating a shape a certain amount, the shape looks the same way it started.

1. Draw the axes of symmetry on the following figures:

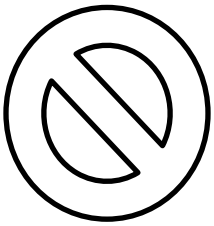


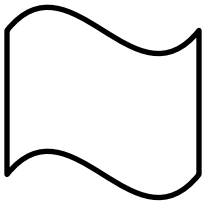


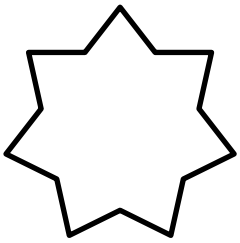


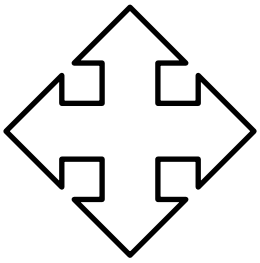


2. Count the number of distinct rotations on the following figures:

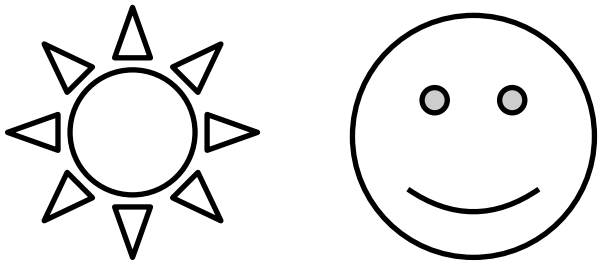








3. Which one of these two shapes has the most symmetry



Describing symmetry via symmetry groups
Normally we think of symmetry as a property of a shape. But Mathematicians focus more on what you can do to a shape and all the ways you can change it while keeping it the same. We want to record all the transformations that leave a shape unchanged.

These transformations are called the “symmetries” of the shape. Taken together they form a “group,” and the relationships between these transformations define the shape’s symmetry.

We describe the relationship between transformations using a Cayley table.
The transformation on the left goes first, followed by the transformation on the top

4. Write the Cayley table for the letter A

*	e	f
e		
f		

5. Write the Cayley table for the letter N

*	e	r
e		
r		

6. Symmetry group of the rectangle

a) Identify the axes of symmetry for the rectangle

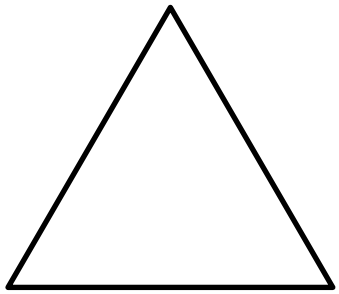
b) Identify the distinct rotations for the rectangle

c) Describe the action of each transformation

d) Write the Cayley table for the rectangle

*	e	f_h	f_v	r
e				
f_h				
f_v				
r				

7. Symmetry group of the triangle



e) Identify the axes of symmetry for the triangle

f) Identify the distinct rotations for the triangle

g) Describe the action of each transformation

h) Write the Cayley table for the triangle

*	e	f_1	f_2	f_3	r_a	r_b
e						
f_1						
f_2						
f_3						
r_a						
r_b						

8. Identify the identity element and the inverse elements of the symmetry groups described in the Cayley tables on questions 4, 5, 6, and 7.

9. Which of the groups on questions 4, 5, 6, or 7 are Abelian?