Activity 3 — Cartesian Products

Remember that for two sets A and B, the *Cartesian product* $A \times B$ is the set of all ordered pairs (a, b) where a is in A and b is in B. Symbolically, this is written:

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

1. What is the Cartesian product of $\mathbb{R}$ and $\mathbb{R}$?

2. Let $A = \{1, 2, 3\}$ and let $B = \{f, g\}$.

   a) Write out all the elements in the set $A \times B$.

   b) Write out all the elements in the set $B \times B$.

3. Let A be the set of solutions to

   $$0 = x^2 - 3x + 2$$

   and let B be the set of solutions to

   $$0 = x^2 - 7x + 10$$

   a) What are A and B?

   b) What is $A \cup B$? $A \cap B$? $A \times B$?

   c) Is $A \times B = B \times A$?

4. Let S^1 be a circle, and let I be an interval (a line segment). The following Cartesian products can be interpreted as shapes. What are the shapes?

   a) $I \times I$

   b) $S^1 \times I$

   c) $S^1 \times S^1$

   d) $I \times I \times I$

Activity 4 — Relations On Sets

1. Take A and B from Problem 3, Activity 3.

   $$A = \{1, 2\} \text{ and } B = \{2, 5\}$$

   Suppose we want to know all the instances when an element of A is less than an element of B. Write out all of these instances as a subset of the Cartesian product.

2. Let A be the set $\{3, 4, 5, 6, 7, 8\}$. Draw graphs depicting the following set relations.

   a) $x \mathbf{R} y \iff x < y$

   b) $x \mathbf{R} y \iff x + y > 11$.

   c) $x \mathbf{R} y \iff 2$ divides $x - y$

3. Let $X = \{a, b, c\}$, and let $\mathscr{P}(X)$ be the power set of B. Define a relation on $\mathscr{P}(X)$ as follows: for all sets A and B in $\mathscr{P}(X)$,

   $$A \mathbf{R} B \iff A \text{ has the same number of elements as } B$$

   Draw a graph depicting this set relation.

4. Let S be the set of all strings of a's and b's of length 3 (for example aab or bab). Define a relation on S as follows: for all strings $s, t \in S$,

   $$s \mathbf{R} t \iff s \text{ and } t \text{ differ in exactly one spot.}$$

   Draw a graph depicting this set relation.