Saturday Morning Math Group
October 27, 2012

“Game Theory and Knowing about Knowledge”

PACKET A

Situation 1

Role: Row Player ("Ralph")

The table below shows your (Ralph's) payoffs:

		Candace: Left	Candace: Right
Ralph	Up	100	100
	Down	0	0

Question: What will you do?

Response:

Situation 2

Role: Column Player ("Candace")

The table below shows your (Candace's) payoffs:

		Candace: Left	Candace: Right
Ralph	Up	3	4
Ralph	Down	-1	2

Question: What will you do?

Response:

Situation 3a

Role: Row Player ("Ralph")

The table below shows your (Ralph's) payoffs:

		Candace: Left	Candace: Right
Ralph	Up	9	0
Ralph	Down	0	9

Question: What will you do?

Response:

Situation 3b

Role: Row Player ("Ralph")

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs):

		Candace: Left	Candace: Right
Ralph	Up	9, 10	0, -2
	Down	0, 3.14	9, 0

Question: What will you do?

Response:

Situation 4a

Role: Column Player ("Candace")

The table below shows your (Candace's) payoffs:

		Candace: Left	Candace: Center	Candace: Right
Ralph	Up	50	100	0
	Middle	200	100	0
	Down	80	90	0

Question: What will you do?

Response:

Situation 4b

Role: Column Player ("Candace")

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs). Recall that I announced that both players see this table.

		Candace		
		Left	Center	Right
Ralph	Up	100, 50	100, 100	0, 0
	Middle	0, 200	0, 100	100, 0
	Down	0, 80	200, 90	50, 0

Question: What will you do?

Response:

Situation 5a

Role: Row Player ("Ralph")

The table below shows your (Ralph's) payoffs:

		Candace: Left	Candace: Right
Ralph	Up	300	0
	Middle	100	100
	Down	0	300

Question: What will you do?

Response:

Situation 5b

Role: Row Player ("Ralph")

The table below shows your (Ralph's) payoffs:

		Candace	
		Left	Right
	Heads	300	0
Ralph	Middle	100	100
	Tails	0	300

Question: What will you do?

Response:

Situation 5c

Role: Row Player ("Ralph")

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs). Recall that I announced that both players see this table.

		Candace: Left	Candace: Right
Ralph	Heads	300, 100	0, 0
	Middle	100, 0	100, 200
	Tails	0, 300	300, 0

Question: What will you do?

Response:

Situation 6

Role: Column Player ("Candace")

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs). Recall that I announced that both players see this table.

		Candace			
		a	*b*	*c*	*d*
Ralph	*A*	3, 2	0, 1	1, 0	0, 0
	B	1, 1	1, 0	1, 1	1, 3
	C	1, 2	0, 4	6, 2	1, 1
	D	0, 4	1, 0	1, 1	3, 3

Question: What will you do?

Response:

Situation 7a

Role: Row Player ("Ralph")

The game is described below. Recall that I announced that both players see this description.

Ralph has two possible actions to choose from: Q and Z.
Candace has two possible actions to choose from: Green and Orange.

If Ralph chooses Q and Candace choose Orange, then both get a payoff of 0.
If Ralph chooses Z and Candace chooses Green, then both get a payoff of 0.
If Ralph chooses Z and Candace chooses Green, then both get a payoff of 10.
If Ralph chooses Q and Candace chooses Orange, then both get a payoff of 10.

Question: What will you do?

Response:

Situation 7b

Role: Row Player ("Ralph")

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs). Recall that I announced that both players see this table.

		Candace: Ft. Awesome	Candace: Loserville
Ralph	Ft. Awesome	10, 10	0, 0
Ralph	Loserville	0, 0	10, 10

Question: What will you do?

Response:

Situation 8

Role: Row Player ("Ralph")

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs). Recall that I announced that both players see this table.

		General Candace	
		Attack	Don't
General Ralph	Attack	3, 3	$-\infty$, 0
	Don't	0, $-\infty$	1, 1

Question: What will you do?

Response:

Situation 9a

Ralph and Candace are each wearing a hat. It is announced to both of them together that 1) each hat is either white or black, and that 2) at least one of the hats is white. Each can see the color of the other's hat, but not the color of his or her own.

Play proceeds in rounds. At the end of each round, each player writes on a slip of paper (that the other cannot see) either "white," "black," or "I don't know." Writing down the correct color of his or her own hat gives a player a payoff of 10, writing down the wrong color gives a payoff of –10, and writing "I don't know" means that the player moves on to the next round. At the end of each round, what each player wrote down is announced to both of them.

Question: How many rounds will the game last? Does it depend on the hat colors?

Response:

Situation 9b

Now Cousin Oliver joins the game.

Again, each player is wearing a hat. It is announced to all of them together that 1) each hat is either white or black, and that 2) at least one of the hats is white. Each can see the color of the others' hats, but not the color of his or her own.

Play proceeds in rounds. At the end of each round, each player writes on a slip of paper (that the other cannot see) either "white," "black," or "I don't know." Writing down the correct color of his or her own hat gives a player a payoff of 10, writing down the wrong color gives a payoff of –10, and writing "I don't know" means that the player moves on to the next round. At the end of each round, what each player wrote down is announced to all of them.

Question: How many rounds will the game last? Does it depend on the hat colors?

Response:

Situation 10

Cousin Oliver goes home, and just Ralph and Candace are left.

A pair of consecutive integers is drawn at random. Ralph sees the even one of the pair, and Candace sees the odd one. (This structure is announced to both of them together.)

A fact X is "common knowledge" between Ralph and Candace if

Ralph knows X.
Candace knows X.
Ralph knows that Candace knows X.
Candace knows that Ralph knows X.
Ralph knows that Candace knows that Ralph knows X.
And so on for such chains *of any length.*

For example, the structure of the game described above is common knowledge, because it was announced to both Ralph and Candace together. (And if something is common knowledge, then the fact that it's common knowledge is itself common knowledge!)

Question: How large must the smaller of the two integers be in order for it to be "common knowledge" that both integers are greater than 0?

Response:

Situation 11

The table below shows both players' payoffs, in the format (Ralph's payoffs, Candace's payoffs).

		General Candace: Attack	General Candace: Don't
General Ralph	Attack	3, 3	$-\infty$, 0
	Don't	0, $-\infty$	1, 1

General Ralph can send a courier to General Candace with a message informing her that he plans to Attack. If General Candace receives such a message, she acknowledges it by sending a message via courier back to General Ralph. If General Ralph receives such a message, he acknowledges it by sending a message via courier back to General Candace. And so on …

At each step, however, there is a 1% chance that the courier gets lost, and the chain of messages stops.

This structure is common knowledge between the two generals.

Question: How many messages must get through for the generals to be able to coordinate on Attacking?

Response: