A new, different world:
Some non-Euclidean geometries

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Geometry

Earth Measuring

Euclid's *Elements*

- ⬠ c. 300 BCE.

- ⬠ Second-most editions of any book, ever.

- ⬠ First axiomatic approach to mathematics.

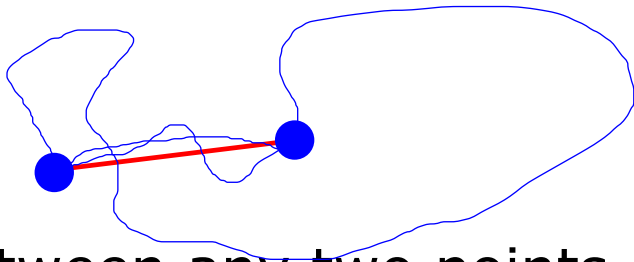- ⬠ Compass and straight-edge constructions

Euclid's postulates

1.

Between any two points...

Euclid's postulates

1.

Between any two points
is a unique line.

Euclid's postulates

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Between any two points
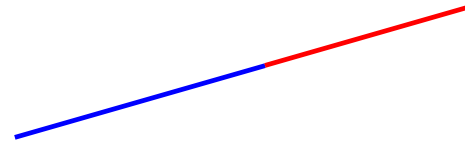is a unique line.

2.

Any line segment...

Euclid's postulates

1.

Between any two points
is a unique line.

2.

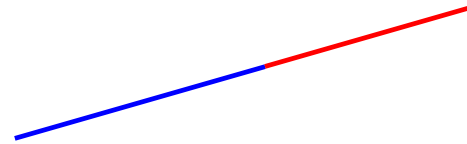Any line segment
can be extended.
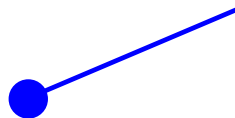
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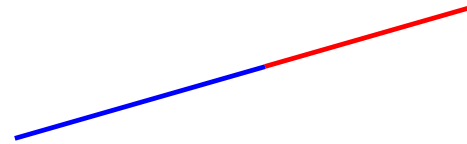
3.

Any point and radius...
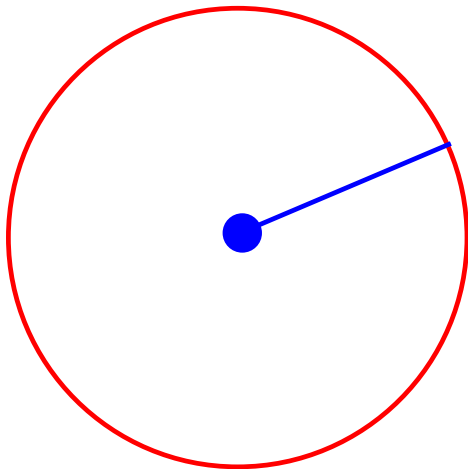
Euclid's postulates

1.

Between any two points is a unique line.

2.

Any line segment can be extended.

3.

Any point and radius define a circle.

Euclid's postulates

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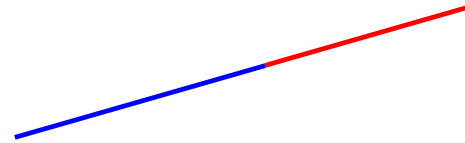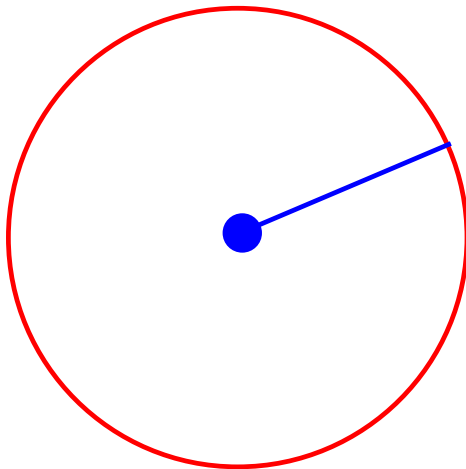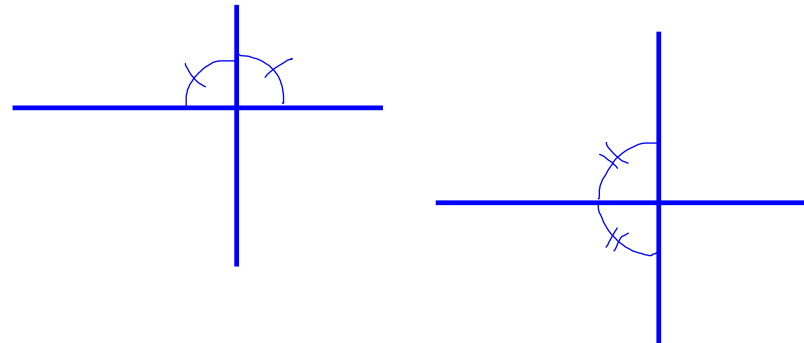define a circle.
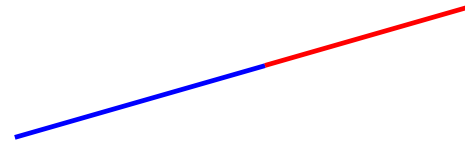
4.

All right angles...

Euclid's postulates

1.

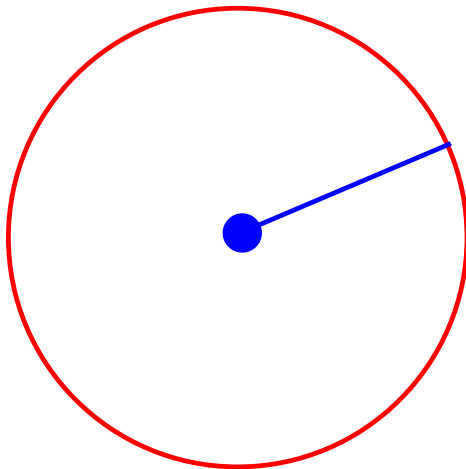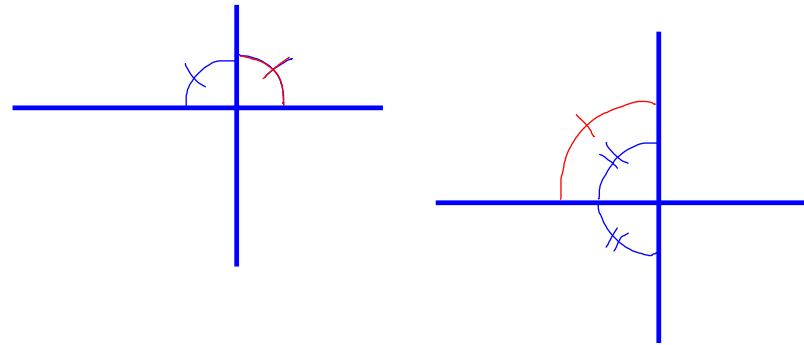Between any two points is a unique line.

2.

Any line segment can be extended.

3.

Any point and radius define a circle.

4.

All right angles are congruent.

Euclid's fifth postulate
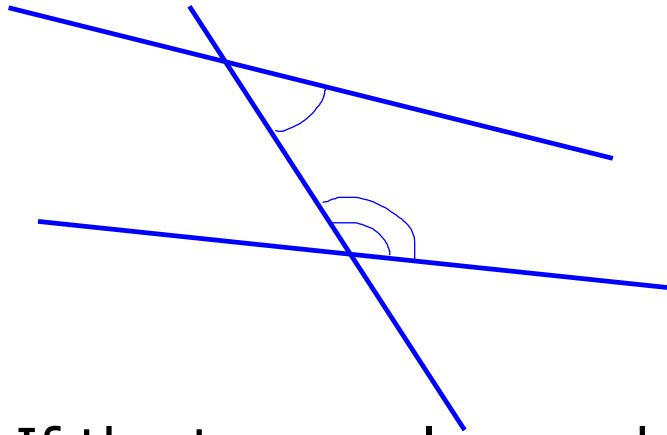
If the two angles marked above sum to less than two right angles...

Euclid's fifth postulate
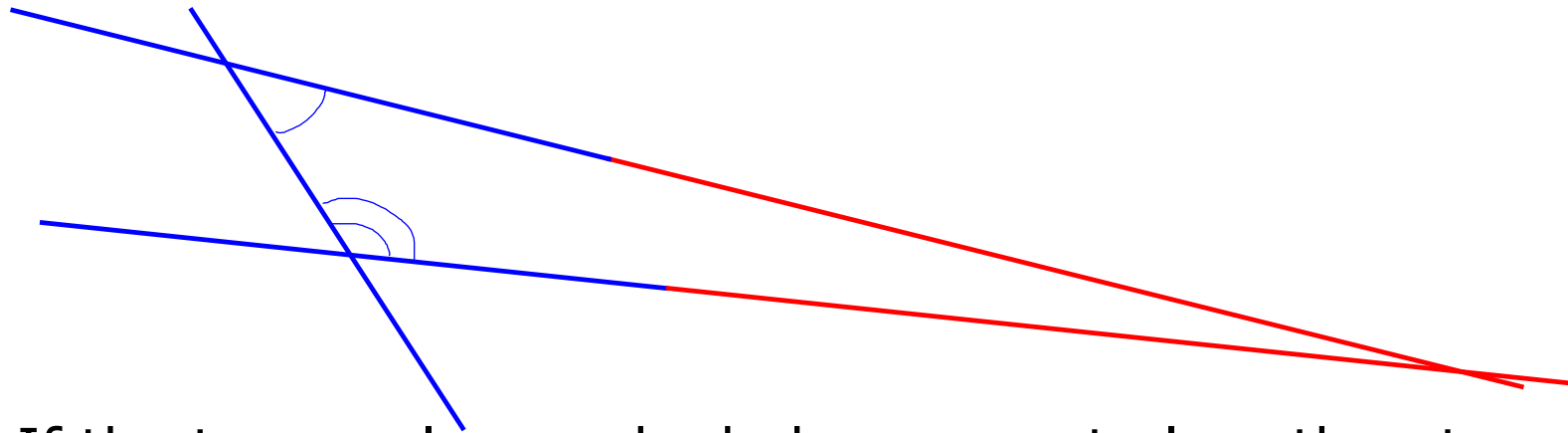
If the two angles marked above sum to less than two right angles, then the lines intersect on that side.

Euclid's fifth postulate
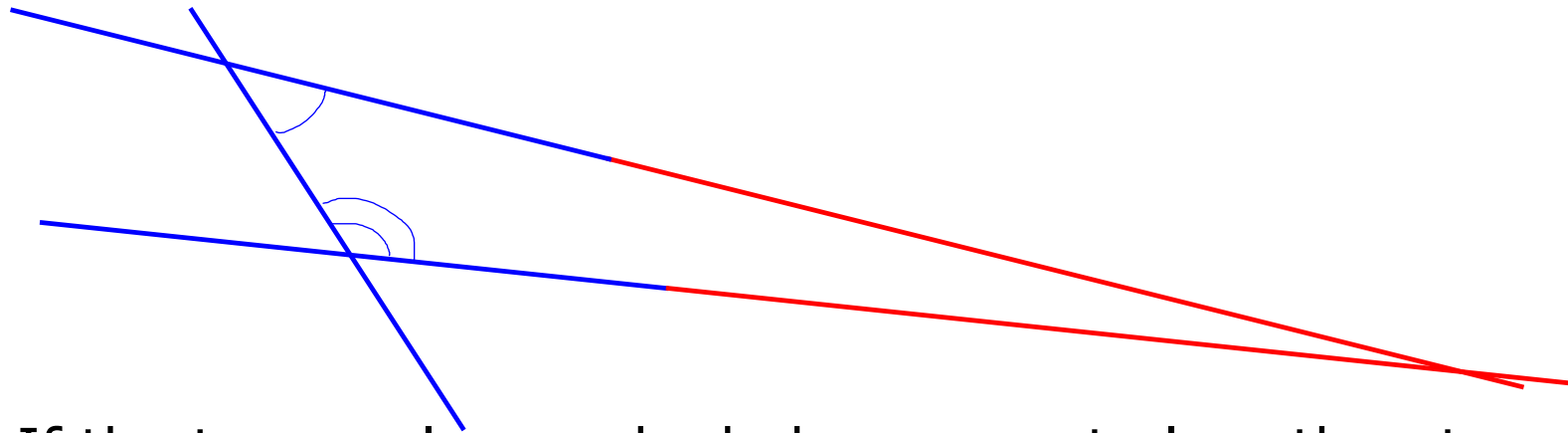
If the two angles marked above sum to less than two right angles, then the lines intersect on that side.

Equivalent statement: given a line and a point not on that line...
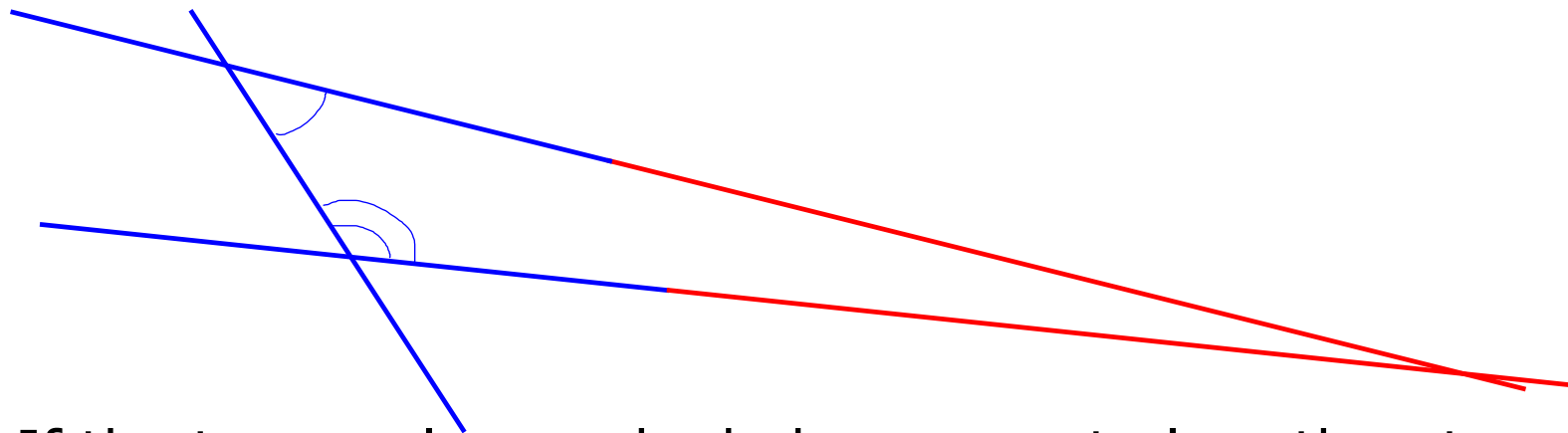
Euclid's Fifth postulate

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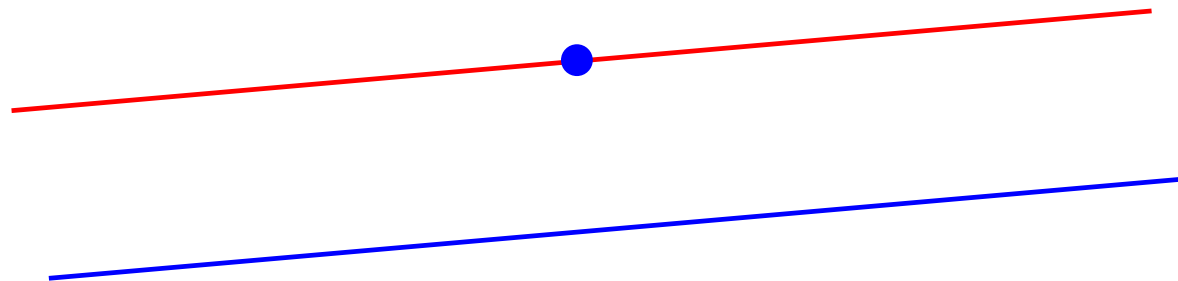
Equivalent statement: given a line and a point not on that line, there is a unique line parallel to the first that passes through the point.

Euclid's Fifth postulate

Mathematicians felt that this postulate ought to be a theorem.

Many attempts at proofs had circular logic: at some point, they used what they were trying to show to justify a step.

Others replaced Euclid's postulate with other (equivalent) statement that they felt were more obvious.

Example (Clairaut, 1741): rectangles exist.

Gauss, Farkas and Janos Bolyai

Farkas to Janos:

You must not attempt this approach to parallels. I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy of my life. ...I have traveled past all reefs of this infernal Dead Sea and have always come back with broken mast and torn sail. The ruin of my disposition and my fall date back to this time. I thoughtlessly risked my life and happiness.

But Janos was trying something different: he assumed this postulate was false.

"Out of nothing I have created a strange new world."

When Farkas sent Janos's work to Gauss, Gauss says that to praise this work

"would amount to praising myself: for the entire content of the work... coincide almost exactly with my own meditations which have occupied my mind from thirty to thirty-five years."

Quotations taken from Euclidean and Non-Euclidean Geormetries: Development and History, 3rd edition, by Martin Jay Greenberg

Wait a minute!

Remember: geometry means measurement of the earth.

What does it mean to actually measure the earth?

Euclid and Bolyai both rule out this model of geometry because the first axiom isn't true here.