Non-Euclidean Geometry

Saturday Morning Math Group September 28, 2013

Euclid's Postulates

You may wish to refer back to these when considering the other two geometry models below.

1. Given any two points, there is exactly one line passing between these points.
2. Any line segment can be extended.
3. Given a point and a distance r, we can construct a circle of radius r around the point.
4. All right angles are congruent. (For Euclid, "right" didn't mean $90°$. It meant an angle that is congruent to its complement, that is, if two congruent angles put side-by-side make a straight line, then they are right angles.)
5. Given a line and a point not on that line, there is a unique line passing through the point that is parallel to the first line.

Spherical Geometry

1. Suppose you want to travel from the north pole of a sphere to another point on the sphere as quickly as possible (and you can't fly or burrow through the sphere–you must stay on the surface). What would the path you take look like? What do we call these lines on the earth?
2. Now suppose that you want to get between any two points on the sphere as quickly as possible. What will such a path look like? You may want to use one of the balls provided to consider this question.
3. I claimed that Euclid's First Postulate is not true on the sphere. Why is this?
4. What does it mean to "extend a line"? Is Euclid's Second Postulate true?
5. What do circles look like? Is Euclid's Third Postulate true? You may with to start by considering circles around the north pole.
6. How should we measure angles on the sphere? Is Euclid's Fourth Postulate true?
7. Is Euclid's Fifth Postulate true on the sphere? Hint: can you find two parallel lines?
8. What do triangles look like? Draw the triangle between the north pole and two points on the equator, one-quarter turn apart. What are the 3 angles in this triangle?
9. Is the Pythagorean Theorem true? (Hint: look back at the previous problem!)

10. In Euclidean geometry, we define π to be the ratio of the circumference to the diameter of any circle–the fact that it doesn't matter what size circle we pick is kind of amazing. In spherical geometry, this doesn't work. Find $\frac{C}{d}$ for two circles centered around the north pole: the equator, and the one halfway between the north pole and the equator. (To do this, make sure to measure the diameter inside the sphere.) You should find that these give two different numbers. What does this tell you about trying to define π on the sphere?

Hyperbolic Geometry

Construct the model of hyperbolic geometry provided. This model was designed by Keith Henderson; the assembly instructions were developed by The Institute for Figuring in conjunction with *Cabinet* magazine.

1. You can lay any pair of polygons flat on the table, and draw a straight line along the polygons and the edges between them (you may want to avoid the vertices for now). Are Euclid's First and Second Postulates true in this model?

2. What should circles look like? Is Euclid's Third Postulate true?

3. Is Euclid's Fourth Postulate true? Make sure to think about what happens at the vertices where three polygons meet. (It might help to know that the measure of an angle in a regular pentagon is $108°$ and in a regular heptagon is $128\frac{4}{7}°$.)

4. Is Euclid's Fifth Postulate true?

5. What do triangles look like in this model? What is the sum of the measures of the angles of a triangle?