1. Use the numbers 2, 3, 4, and 5 all *exactly* once and the + sign as many times as you want to make a true equation without using *any other symbols* (except for the equals sign!)

2. You have two wooden cubes and a marker. You are allowed to write any *digit* (meaning 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) on each of the six faces of each cube. Your job is to paint the blocks so that you can display the numbers 01, 02, 03, 04, 05, all the way through 31 (to make a calendar). How can you do this if you are allowed to use the fact that 6 is 9 "up side down?" Can you do this if you aren't allowed to use this fact?

3. Teddy has three white frisbees and three black frisbees. When Teddy comes home one day, he sees his roommate Richard and explains that he has put two white frisbees in one box, put two black frisbees in another box, and put the remaining two frisbees in a third box. He then explains that he labeled the boxes "2 White Frisbees," "2 Black Frisbees," and "1 White and 1 Black Frisbee," but he tells Teddy that each label on the box is *wrong.*

   Richard then explains that he is only going to allow Teddy to pick *one* box of the three of them and only take *one* frisbee out at random, without looking. How can Teddy figure out which box is which?

4. On an island far far away, there are two types of people--*truthers,* those people who can only say things they know for a 100% fact are true, and *liars,* those people who can only say things they know for a 100% fact are false.

   You're treasure hunting on the island and you find two doors guarded by two people that you know live on the island. You know one of them is a truther and one of them is a liar. They are each guarding one door--one of which has treasure behind is and one of them has a person eating tiger. The problem is, you don't know who is guarding what door! You pick one of the people at random (and you don't know whether they're the liar or truther). What's a question you can ask to figure out which door has the treasure?

5. Using the numbers 1,3,4, and 6 all *exactly* once and the symbols +, -, *, /, (, and) as many times as you want, create the number 24.

6. (Hat Puzzle) A guard plays a game with prisoners in a prison. Each prisoner is assigned a random hat, either red or blue, but the number of each color hat is not known to the prisoners. The prisoners will be lined up single file where each can see the hats in front of him but not behind. Starting with the prisoner in the back of the line and moving forward, they must each, in turn, say only one word which must be "red" or "blue". If the word matches their hat color they are released, if not, they are killed on the spot. A sympathetic guard warns them of this test one hour beforehand and tells them that they can formulate a plan where by following the stated rules, 9 of the 10 prisoners will definitely survive, and 1 has a 50/50 chance of survival. What is the plan?

7. Two people play a game of poker in a strange way. They spread a deck of 52 cards (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, each of one of four suits --hearts, spaces, diamonds, or clubs) so that they can see all the cards. The first player draws a hand by picking any five cards he wants. The second player does the same. The first player now may keep his original hand or draw up to five cards and discard the same number of cards from his hand. The discards are put outside of the game.

   The second player may now draw in the same way. The person with the highest hand wins, but don't forget the suits have equal values, so straights, flushes, and straight flushes tie unless one of the players has higher cards.

   After a while of playing this game, the first player discovers that they can always win this game! How can they do this?

Ordering of Poker Hands - *Please ask a volunteer if this isn't clear!*

The best hands are straight flushes (where the highest card wins), then four of a kind (having four of the same value, like J♦ J♣ J♥ J♠ Q♠), then the next best hands are full houses, then flushes, then straights.

A *full house* is a hand that contains three cards of one rank and two cards of another rank, such as 3♣ 3♠ 3♦ 6♣ 6♥

A *straight* is a hand of five cards that is in order such as 7♣ 6♠ 5♠ 4♥ 3♥ (a "seven-high straight"). An ace can rank either high (as in A♦ K♣ Q♣ J♦ 10♠, an ace-high straight) or low (as in 5♣ 4♦ 3♥ 2♥ A♠, a five-high straight), but cannot simultaneously rank both high and low, so Q♠ K♠ A♣ 2♥ 3♦ is not a straight).

A *flush* is a hand that contains five cards of the same suit, and a *straight flush* is a flush that is also a straight, like A♦ K♦ Q♦ J♦ 10♦.

8. You meet three people on the island. You know one is a tourist, one is a liar, and one is a truther, but you don't know who is who! (A *truther* is someone who can only say true things, a *liar* is someone who can only say false things, and a *tourist* is someone who randomly switches between saying true things or lies and you can't tell which is which.)

   You pick someone at random and get to ask one (and only one) question. What one question can you ask if your goal is to find someone who lives on the island? (Note: You don't have to know if they're a liar or truther, just that they aren't a tourist.)

9. (Putnam 2001 A1) Let * be a *binary operation* on some set S--this means that it takes in two (not necessarily different!) elements of S, call them a and b, and returns an element of S that we write a*b. Let * be a binary operation on some set S such that (a*b)*a = b for all a and b (not necessarily different!) in S. Prove that for all a and b (not necessarily different!) that a * (b * a) = b.

10. You have a broken lock which takes three numbers between 1 and 8. However, this padlock is broken--if you guess two of the digits in the right spot, the lock will open! What is the smallest number of codes you need to try to guarantee you break the lock?

(Note that there are lots of ways to do this with 64 test combinations. However, each combination you test covers 22 possible cases, and there are only 8^3 = 512 possible combinations in all, so in theory you might be able to get away with 24 ≈ 512/22 test combiations. The truth, then, lies in the middle--but where?)