1. You have a 1 pound weight, a 2 pound weight, and a 4 pound weight. Can you combine the weights to make something that weighs 5 pounds? Can you combine the weights to make something that weighs 6 pounds?

2. What positive integers (meaning numbers on the list 1,2,3,4,5,...) can you make as weights if you have a 1 pound weight, a 2 pound weight, and a 4 pound weight? Are there any different collections of weights that combine to have the same weight?

3. What positive integers (meaning numbers on the list 1,2,3,4,5,...) can you make as weights if you have a 1 pound weight, a 2 pound weight, a 4 pound weight, and an 8 pound weight? Are there any different collections of weights that combine to have the same weight?

4. What positive integers (meaning numbers on the list 1,2,3,4,5,...) can you make as weights if you have a 1 pound weight, a 2 pound weight, a 4 pound weight, a 8 pound weight, and a 16 pound weight?

5. We noticed that we could write 19 = 16 + 2 + 1. We say the *binary representation* of 19 is 10011_2, where the "2" at the bottom reminds us that our weights are $1=2^0$, $2=2^1$, $4=2^2$, $8=2^3$, $16=2^4$, and so on. Another example is $26 = 11010_2$, because $26 = 2^4 + 2^3 + 2^1$. What number is 10111_2?

6. What number is 1101_2? How about 1010_2? (*Example:* $1110_2 = 2^3 + 2^2 + 2^1$)

7. What number is 110001_2? How about 111111_2?

8. Write the numbers 7, 10, 31, and 34 in "base 2" (meaning find the binary representation of these numbers). If you're given the binary representation of a number, how can you tell, right away, whether it is even or not?

9. You have a 1 pound weight, a 3 pound weight, and a 9 pound weight. Can you combine these to get every weight between 1 and 13?

10. What if you have *two* one pound weights, *two* 3 pound weights, and *two* 9 pound weights? Convince a volunteer that you could make any weight from 1 to 26 (but you don't have to write out all 26 numbers!).

11. We noticed that we could write $19 = 2*3^2 + 0*3^1 + 1$. We say the *base 3 representation* of 19 is 201_3, where the "3" at the bottom reminds us that our weights are $1=3^0$, $3=3^1$, $9=3^2$ and so on. Another example is $15 = 120_3$, because $15 = 1*3^2 + 2*3^1 + 0$. What number is 2001_3? What number is 122_3?

12. What is the base 3 representation of 26?

13. What number is 83_9? (Here, we're using the base of 9. Try to generalize what you've learned from base two and three to guess what number this is. If you're stuck, ask a volunteer for help!)

14. What base are the numbers we use to count in? (If you're stuck, ask a volunteer for help!)

15. What is $1010101_3 + 1011_3$? Try to do this without converting back into base 10.

16. What is $1111_2 + 1000_2$? Try to do this without converting back into base 10.

17. How can you tell just by looking at the binary representation of a number if that number is divisible by 4?

18. The first water lily in the Lily Pond blossomed on June 1st. Since that day, the number of flowers has doubled every day. On June 15th, the flowers covered the pond completely. If two flowers (instead of one) started blooming on June 1st and the number of flowers doubled every day, when would the pond become completely covered in flowers?

19. To get to the Stone of Wisdom, a young wizard must solve a puzzle. She is given a 7×7 square that she has to cut into 9 smaller rectangular pieces. Using these pieces, she should be able to tile any integer rectangle that fits into the 7 × 7 square: 1 × 1, 1 × 2, 1 × 3,. . . , 1 × 7, 2 × 2, 2 × 3,. . . , 7 × 6, and 7 × 7. Solve this puzzle. (All cuts should follow grid lines. “To tile” means to cover without gaps and overlaps.)

20. What is the binary representation of $64 * 111_2$? Note that 64 is written in base 10, while 111_2 is written in base 2. (*Hint:* $64 = 2^6$.)

21. What is the binary representation of $65*111_2$? Note that 65 is written in base 10, while 111_2 is written in base 2. (*Hint:* $64 = 2^6$. How can you use this to convert 65 to binary?)

22. Merlin the Sorcerer has built his castle next to the dwelling place of the mighty Dragon Shmok. Every morning with the first rays of the rising sun the Dragon flies out of his lair, heading toward one of the four nearby towns, which are located to the north, south, east, and west of the castle. Merlin’s task is to warn the townspeople where to expect Shmok today.

The castle has three towers topped with magic spheres; one is made out of ruby, the other of emerald, and the third one is made out of sapphire. When the magic spheres are lit up, their radiance is seen from the farthest corners of the land. So it has been agreed that by these lights Marlin sends his signal, using some code to indicate where the Dragon is heading.

The spheres’ lights, however, tend to flicker randomly at night, and the morning may find any combination of them turned on or off. And, because of his old age, Merlin will have the strength to climb only one tower to manipulate its light (to turn it on or off).

Thus, starting from a random combination, Merlin can change the state of no more than one sphere. After that, the townspeople should be able to read where the Dragon is heading by seeing which lights are on.

Can you help Merlin come up with such a code?

23. If you write the integers from 1 to 256 in binary, how many zeroes do you write?