

Homework 1

due Thursday, September 5, 11:00

Problem 1. Show that

$$\sum_{k=1}^n k^2 = \frac{n(2n+1)(n+1)}{6}$$

for every natural number $n \in \mathbb{N}$.

Problem 2. Show that $2^n \geq n^2$ for every integer $n \geq 4$.

Problem 3. Show that, for every integer $a \in \mathbb{Z}$, $a^3 - a$ is divisible by 3.

Problem 4. Using only the axioms of the integers, show that

$$(-a) \cdot (-b) = ab$$

for all integers $a, b \in \mathbb{Z}$.

Problem 5. Let $a, b, c \in \mathbb{Z}$ such that $a > b$ and $b > c$. Show, using only the axioms of the integers, that $a > c$.