Homework 3

due Thursday, September 19, 11:00

Problem 1. How many integer solutions $x, y \in \mathbb{Z}$ do the following equations have? If there is at least one solution, find an example.

a) $60x + 18y = 97$

b) $37x + 1000010000100001y = 0$

c) $14541x + 1367631y = 13566531$

Problem 2. Let a, b be positive integers such that $a^2 \mid b^2$. Show that $a \mid b$.

Problem 3. Denote by p_n be the n–th prime ($p_1 = 2, p_2 = 3, p_3 = 5, \dots$). Show that $p_n \leq 2^{2^{n-1}}$ for every $n \in \mathbb{N}$. Conclude that there are at least $n + 1$ primes less than 2^{2^n} for every $n \in \mathbb{N}$.

Hint: Look at Euclid's proof of the infinitude of primes.

Problem 4. Let $p \neq q$ be prime numbers and a be an integer such that $p \mid a$ and $q \mid a$. Show that $pq \mid a$. Find a counterexample if either p or q is not prime.

Problem 5. An integer a is called an *n–th power* for $n \in \mathbb{N}$ if there exists an integer b such that $a = b^n$.

For two different prime numbers p and q show that a positive integer a is a p–th power if and only if a^q is a p–th power.