Homework 8

due Tuesday, October 29, 11:00

Problem 1. Let $m \in \mathbb{N}$, $m > 2$, and let $\mathbb{Z}_m^\times = \{\overline{a} \mid (a, m) = 1\} \subset \mathbb{Z}_m$ be the subset of elements of $\mathbb{Z}_m$ which are coprime to m. Show that

$$\sum_{x \in \mathbb{Z}_m^\times} x = \overline{0}$$

Problem 2. Find $(n! + 1, (n + 1)!)$ for $n \in \mathbb{N}$.

Problem 3. Let p be a prime and $a, b \in \mathbb{Z}_p$. Show that $(a + b)^p = a^p + b^p$.

Problem 4. For a prime number p and $m \in \mathbb{N}$ let $\nu_p(m)$ be the exponent of p in the prime–power factorization of m (for example $\nu_3(5) = 0, \nu_3(12) = 1, \nu_3(54) = 3$). Prove that

$$\nu_p(n!) = \sum_{i=1}^{\infty} \left\lfloor \frac{n}{p^i} \right\rfloor,$$

where $\lfloor - \rfloor$ is the floor function from Homework 2.