Fall 2021 – Math 328K – 55385

Homework 2

due Tuesday, September 14, 14:00

Problem 1. Show that, $3 \mid a^3 - a$ for every $a \in \mathbb{Z}$.

Problem 2. The *greatest integer function* or *floor function* $\lfloor \cdot \rfloor \colon \mathbb{R} \to \mathbb{Z}$ is defined as follows: for every real number $x \in \mathbb{R}$, $\lfloor x \rfloor$ is the greatest integer which is less or equal to x.

Using the floor function and the basic arithmetic operations, find an explicit formula for the quotient and remainder of two integers. Prove your result.

Problem 3. Suppose that $a, b \in \mathbb{Z}$ and $a \mid b$. Show that $a^n \mid b^n$ for every $n \in \mathbb{N}$.

Problem 4. The following are meant to help you avoid common mistakes.

(a) Find integers a, b and c such that $a \mid bc$ but $a \nmid b$ and $a \nmid c$.

(b) Find integers a, b and c such that each pair of them has a common divisor greater than 1, but that all three of them together do not have a common divisor greater than 1.

Problem 5. Let a and b be integers. Recall that a *pair of Bezout coefficients* for a and b is a pair of integers $m, n \in \mathbb{Z}$ such that

$$ma + nb = (a, b).$$

Prove that, for any fixed pair of integers a and b, there are infinitely many pairs of Bezout coefficients.