Fall 2021 – Math 328K (55385)

Practice midterm exam

Problem 1. Define prime numbers and composite numbers.

Problem 2. What is the last decimal digit of the number 4^{44}?

Problem 3. The *Fermat numbers* F_n are defined by

$$F_n = 2^{2^n} + 1$$

for all non–negative integers n.

a) Show that $F_{n+1} = F_0 F_1 \cdots F_n + 2$ for all non–negative integers n.

b) Show that F_n and F_m are coprime, for any integers $0 \leq n < m$.

Problem 4. Do the following equations have solutions x in $\mathbb{Z}/96\mathbb{Z}$? How many? If there are solutions, list all of them.

a) $9x = 5$

b) $5x = 9$

Problem 5. Let $a_1, \ldots, a_n, b \in \mathbb{N}$ be positive integers and let $A = a_1 a_2 \cdots a_n$. Show that if $(a_i, b) = 1$ for all $i \in \{1, \ldots, n\}$, then $(A, b) = 1$.