Spring 2020 – Math 367K – 53115

# Exercise Sheet 3

*due Thursday, February 13, 9:30*

**Exercise 1.** Let $X, Y$ be topological spaces and $A \subset X$, $B \subset Y$ subsets. Show that if you take the product topology on $X \times Y$ and then the subspace topology on $A \times B \subset X \times Y$ you obtain the same topology as if you first take the subspace topologies on $A$ and $B$ and then the product topology on $A \times B$.

**Exercise 2.** Let $X$ be a topological space and $A, B \subset X$. Show that

a) if $A \subset B$, then $\overline{A} \subset \overline{B}$.

b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

c) $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$

d) Find a counterexample for $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

**Exercise 3.** Let $X$ be a topological space. Show that $X$ is Hausdorff if and only if the diagonal

$$\Delta = \{(x, x) \mid x \in X\}$$

is closed in the product topology on $X \times X$.

**Exercise 4.** Recall the topologies on $\mathbb{R}$ from the last homework:

$$\begin{aligned}
\mathcal{T}_2 &= \text{the finite complement topology} \\
\mathcal{T}_3 &= \text{the lower limit topology, having all sets } [a, b) \text{ as basis} \\
\mathcal{T}_4 &= \text{the upper limit topology, having all sets } (a, b] \text{ as basis} \\
\mathcal{T}_5 &= \text{the topology having all sets } (-\infty, a) \text{ as basis}
\end{aligned}$$

Describe for each of them which sequences converge to which limits.

**Exercise 5.** Show that the intersection of dense sets need not be dense, but the intersection of two open dense sets is always open and dense.