0. Functions, differentiation and integration

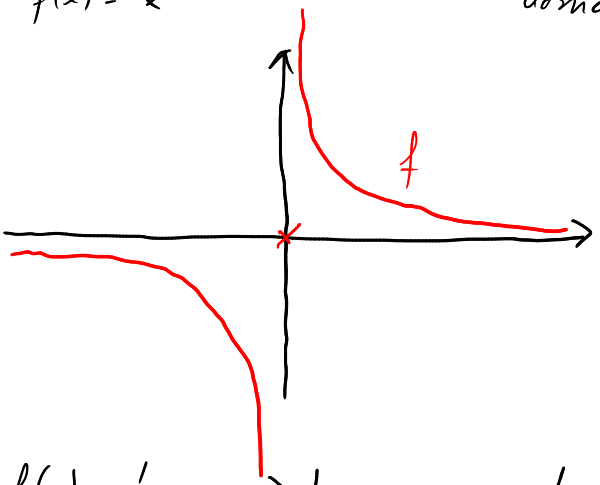
A function f is a rule which assigns to each x in its domain D a value $f(x)$ in its codomain E.

- For us, E will always be the set of real numbers $\mathbb{R}$ and D (almost) always a subset of $\mathbb{R}$.

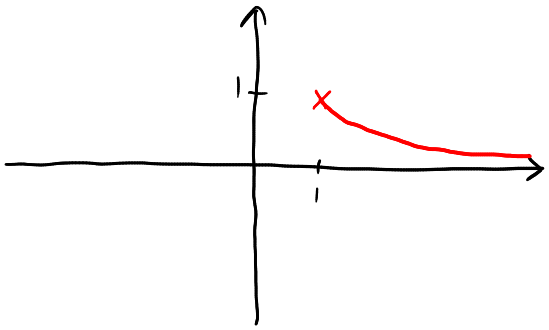
Notation / examples: difference $A \setminus B$, union $A \cup B$, intersection $A \cap B$

Convention: If no domain is given, it's the largest possible one.

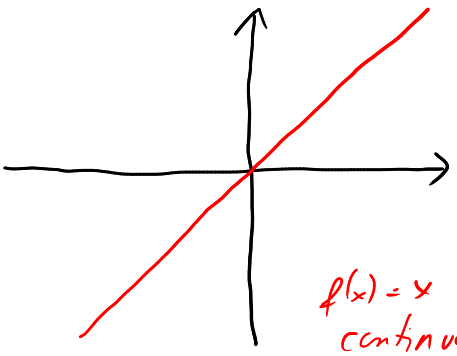
- $f(x) = \frac{1}{x}$ domain $= \mathbb{R} \setminus \{0\}$



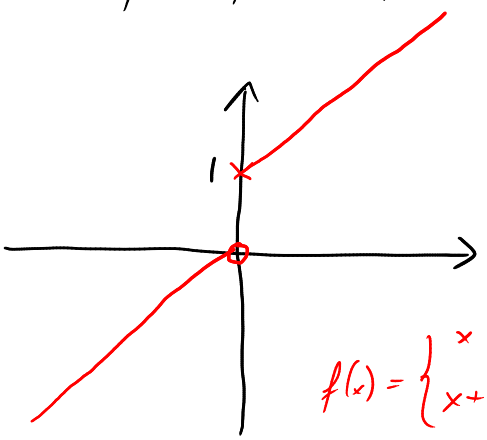
- $f(x) = \frac{1}{x}$, $x \geq 1$ domain $= [1, \infty)$



- A function f is continuous if $\lim_{x \to a} f(x) = f(a)$ for every a in the domain of f.



$f(x) = x$ continuous

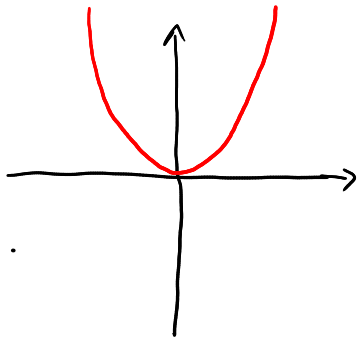


$$f(x) = \begin{cases} x & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$$

- f is <u>differentiable</u> if

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

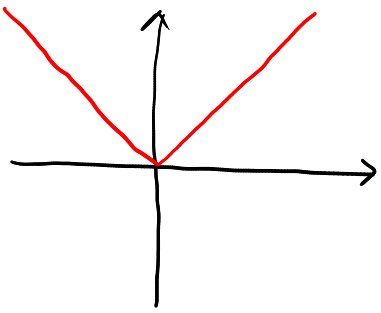
exists for all x in the domain of f. f' is the <u>derivative</u> of f.



$f(x) = x^2$

differentiable

$f'(x) = 2x$



$f(x) = |x|$

not differentiable

$$f'(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

$f'(0)$ is undefined

$$f'(0) = \lim_{h \to 0} \frac{|h|}{h}$$

does not exist.

<u>Differentiation rules</u>

$f(x)$	$f'(x)$
x	1
7	0
ax^n	anx^{n-1}
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$g(x)h(x)$	$g'(x)h(x) + g(x)h'(x)$
$g(h(x))$	$g'(h(x))h'(x)$
$\frac{g(x)}{h(x)}$	$\frac{g'(x)h(x) - h'(x)g(x)}{h(x)^2}$
$g^{-1}(x)$	$\frac{1}{g'(g^{-1}(x))}$
$\frac{1}{g(x)}$	$-\frac{g'(x)}{g(x)^2}$