Logarithmic Properties and Equations

OBJECTIVES:

1. Know the fundamental properties of logarithms.
2. Use the fundamental properties of the logarithm to solve equations.
3. Know the Product Property, Quotient Property, and Power Property of logarithms.
4. Be able to simplify and expand expressions by using the properties of logarithms.
5. Be able to solve logarithmic equations using the uniqueness property.
6. Solve application problems to all the concepts above.

Fundamental Properties of Logarithms.

State the four fundamental properties of Logarithms and an explanation on why they are true.

I.

II.

III.

IV.

Use the properties to solve the following problems:

1. $\log x = 4$
2. $10^{2x} + 15 = 100$
3. $2\ln(x + 2) - 4 = 3$
4. $3e^{x-1} - 3 = 4$

State the following properties of logarithms and provide a proof in your own words.

I. The Product Property

II. The Quotient Property

III. The Power Property

IV. The Uniqueness Property

Simplifying Expressions using properties of logs

Rewrite the following as one logarithm using the properties of logs

1. $\log x + \log 2$

2. $\ln(x-1) - \ln(x+2)$

3. $\log_3(x) + \log_3(x-1) - 2\log_3(2x-1)$

Expand the following into sums and products using the properties of logs.

1. $\log_2(xy^2)$

2. $\ln(\sqrt{4x})$

3. $\ln(ex\sqrt{z})$

Solve the following equations

1. $\log(x+12)+\log(x-2)=7$

2. $\ln(x)-\ln(x+1)=3$

3. $\log(-x-1)=\log(5x)-\log(x)$

4. $\ln 6-\ln(5-r)=\ln(r+2)$

5. $4^{2x-1}=7^{3x}$

6. $e^{2x}-3e^{x}+2$

7. $\log_2(x+5)=\log_4(3x)$

INTERESTING PROBLEMS

1. The number of toy planes an employee can assemble from its component parts depends on the length of time the employee has been working. This output is modeled by $P(t) = 5.9 + 12.6 \ln t$, where $P(t)$ is the number of planes assembled daily after working t days. (page 395, 127)

   (a) How many planes is an employee making 5 days on the job?

   (b) How many days until the employee is able to assemble 34 planes per day?

2. Show the following are false by finding a counterexample: (page 396, 139)

   (a) $\ln(pq) = \ln(p)\ln(q)$

   (b) $\ln \frac{p}{q} = \frac{\ln p}{\ln q}$

   (c) $\ln p + \ln q = \ln(p + q)$

3. Find the zeros of the following functions

   (a) $f(x) = \ln(x + 1) - \ln(x - 2)$

   (b) $f(x) = \log(2x) + \log(x - 4) - 3$

   (c) $g(x) = 3 + \ln(6) - \ln(2x + 1)$

4. After expanding their area of operations, a manufacturer of small storage buildings believes the larger area can support sales of 40 units per month. After increasing the advertising budget and enlarging the sales force, sales are expected to grow according to the model $S(t) = \dfrac{40}{1 + 1.5e^{-0.08t}}$, where $S(t)$ is the expected number of sales after t months. (page 394, 116)

   (a) How many sales were being made each month, prior to the expansion?

   (b) How many months until sales reach 25 units per month?

5. If $f(x) = e^{x+1} - 5$, what is $f^{-1}(x)$? Provide the graph of both.

NOTES: