PDE I – HOMEWORK ASSIGNMENT 3

Due Monday, September 20, 2010. **Please write clearly, and staple your work !**

1. Problem

Consider the wave equation $u_{tt} - \Delta u = 0$ on $\mathbb{R}^n \times \mathbb{R}_+$, with initial data $u = g$, $u_t = h$ on $\mathbb{R}^n \times \{0\}$, where $g \in C^m(\mathbb{R}^n)$ and $h \in C^{m-1}(\mathbb{R}^n)$, for $m = \frac{n+1}{2}$ if n is odd, and $m = \frac{n+2}{2}$ if n is even.

(a) Solve it for dimension $n = 5$, using the method of spherical means.

(b) Solve it for dimension $n = 4$, using the method of descent.

(c) Show that $u \in C^2(\mathbb{R}^n \times \mathbb{R}_+)$.

2. Problem

Consider the same homogenous wave equation as above, but in dimension $n = 3$. Moreover, assume now that g and h are smooth and have compact supports. Prove that there exists a constant C such that $|u(x,t)| \leq C/t$, for every $x \in \mathbb{R}^3$, and for $t > 0$.

3. Problem

Assume that $u \in C^2(\mathbb{R} \times \mathbb{R}_+)$ solves the homogenous wave equation in dimension $n = 1$, with initial data $u = g$, $u_t = h$ at $t = 0$, where both g and h are smooth with compact supports. Define the kinetic energy

$$K(t) := \frac{1}{2} \int_{\mathbb{R}} u_t^2(x,t)dx$$

and the potential energy

$$P(t) := \frac{1}{2} \int_{\mathbb{R}} u_x^2(x,t)dx\,.$$

Prove that:

(a) $K(t) + P(t)$ is constant in time t.

(b) $K(t) = P(t)$ for all sufficiently large times t (equipartition of energy).