PDE I – HOMEWORK ASSIGNMENT 8

Due Monday, November 1, 2010. **Please write clearly, and staple your work !**

1. Problem

Let $k_{ij}(x) := \frac{x_i x_j}{|x|^{n+2}}$, for $i, j \in \{1, \dots, n\}$, and $i \neq j$. Prove that

$$T_{ij} f(x) = \int k_{ij}(x-y) f(y) dy$$

is a Calderon-Zygmund operator.

2. Problem

For which values of $q = q(p, r)$ can the inequality

$$\|k * f\|_q \leq C \|k\|_r \|f\|_p$$

possibly be satisfied for all $k \in L^r(\mathbb{R}^n)$ and $f \in L^p(\mathbb{R}^n)$ with $1 \leq p, r \leq \infty$? Determine $q = q(p, r)$ and prove the inequality.

3. Problem

Let $s > 0$. Define the Sobolev space $H^s = \{f \in L^2(\mathbb{R}^n) \mid \|f\|_{H^s} < \infty\}$ where

$$\|f\|_{H^s}^2 = \int_{\mathbb{R}^n} (1 + |\xi|^2)^s |\widehat{f}(\xi)|^2 d\xi \, .$$

Prove that for $s > \frac{n}{2}$, $H^s(\mathbb{R}^n)$ embeds in the space of bounded continuous functions. That is, $\|f\|_{C^0(\mathbb{R}^n)} \leq C \|f\|_{H^s}$.