Ex True or false:

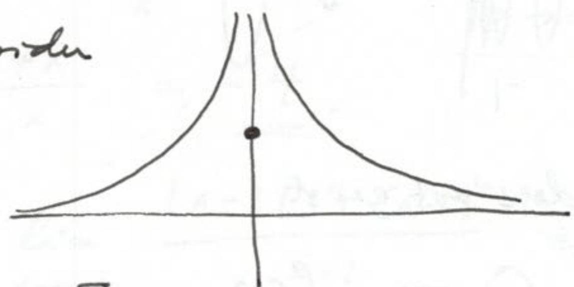
if $\lim_{x \to 6} (f(x) \cdot g(x))$ exists, then it must be $= f(6) \cdot g(6)$.

limit, equal only if continuous.

Ex T or F:

if the line $x = 0$ is a vertical asymptote of $f(x)$, then f is not defined at $x = 0$.

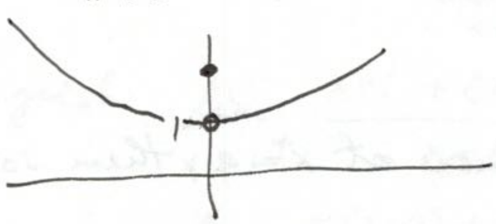
consider



$$f(x) = \begin{cases} \frac{1}{x^2}, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0. \end{cases}$$

Ex T or F:

if $f(x) > 1$ for all x, and $\lim_{x \to 0} f(x)$ exists, then $\lim_{x \to 0} f(x) > 1$.



consider $f(x) = \begin{cases} 1 + x^2, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$

$\Rightarrow f(x) > 1$ for all x.

$\lim_{x \to 0} f(x) = 1$

Ex if $p(x)$ is a polynomial, then $\lim_{x \to a} p(x) = p(a)$

T or F ?

because polynomials are continuous for all x.

Ex $$\lim_{x\to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1}}{x} = \lim_{x\to 0} \frac{\sqrt[3]{1+x} - 1}{x}$$

$$t := \sqrt[3]{1+x} \quad , \quad t \to 1$$

$$t^3 = 1 + x \Rightarrow x = t^3 - 1$$

$$\Rightarrow \dots = \lim_{t\to 1} \frac{t-1}{t^3-1} = \lim_{t\to 1} \frac{\cancel{t-1}}{\cancel{(t-1)}(t^2+t+1)} = \underline{\underline{\frac{1}{3}}}.$$

<u>Note:</u> $t^3 - 1 = (t-1)(t^2+t+1)$.

$$t^n - 1 = (t-1)(t^{n-1} + t^{n-2} + \dots + t + 1).$$

Another way to do it:

$$\lim_{x\to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1}}{x}$$

$$f(t) = \sqrt[3]{t} = t^{1/3}$$

$$f'(t) = \frac{1}{3} t^{-2/3}.$$

$$= \lim_{h\to 0} \frac{\sqrt[3]{1+h} - \sqrt[3]{1}}{h}$$

$$= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = f'(1).$$

$$= \frac{1}{3} \cdot (1)^{-2/3} = \underline{\underline{\frac{1}{3}}}$$

$$1^{-2/3} = \frac{1}{\underbrace{1^{2/3}}_{1}}$$

$$(x^r)' = r x^{r-1}$$

Ex $\lim_{x \to 0} \dfrac{\sqrt[4]{2+x} - \sqrt[4]{2}}{2x}$. $\qquad f(x) = \sqrt[4]{x} = x^{1/4}$

$= \dfrac{1}{2} \lim_{h \to 0} \dfrac{\sqrt[4]{2+h} - \sqrt[4]{2}}{h} \qquad f'(x) = \dfrac{1}{4} x^{-3/4}$

$= \dfrac{1}{2} f'(2) = \dfrac{1}{2} \cdot \dfrac{1}{4} \cdot 2^{-3/4} = \dfrac{1}{8} \dfrac{1}{2^{3/4}}$

Ex $f(x) = \dfrac{3x^3 + 2x^2 + x + 1}{x^2 + 2}$.

$f'(x) \underset{\uparrow}{=} \dfrac{(9x^2 + 4x + 1) \cdot (x^2+2) - (3x^3 + 2x^2 + x + 1) \cdot (2x)}{(x^2+2)^2}$.

quotient rule.

Ex $f(x) = x \cdot \ln x$

$f'(x) \underset{\uparrow}{=} 1 \cdot \ln x + x \cdot \dfrac{1}{x} = \ln x + 1$.

product rule.

Ex T or F:

f is continuous on $[-1, 1]$, $f(-1) = 4$ and $f(1) = 3$.
Then, there exists a number r between -1 and 1, such that $f(r) = \pi$.

intermediate value thm: f continuous.

$3 < \pi < 4$.

Ex Distance of a space ship from the earth at t seconds is

$$s(t) = t^4 + t^2 \text{ meters}.$$

What is its instantaneous velocity after 10 sec?

$$v(t) = s'(t) = 4t^3 + 2t .$$

$$v(10) = 4000 + 20 = 4020 \text{ m/sec}$$

Acceleration

$$a(t) = v'(t) = s''(t)$$

$$= 12t^2 + 2 .$$

$$a(10) = 1200 + 2 = 1202 \text{ m/sec}^2$$

Average velocity between 5 sec and 10 sec:

$$\overline{v} = \frac{s(10) - s(5)}{10 - 5} = \frac{10^4 + 10^2 - 5^4 - 5^2}{5}$$

$$\overline{a} = \frac{v(10) - v(5)}{10 - 5} = \ldots$$