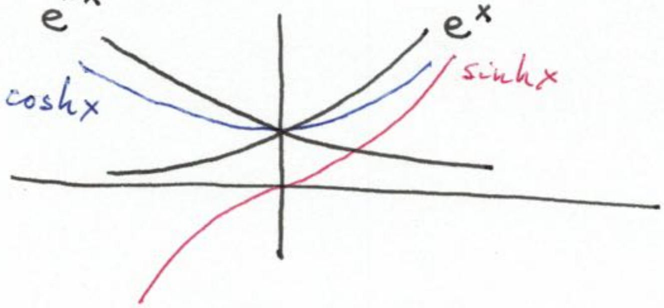
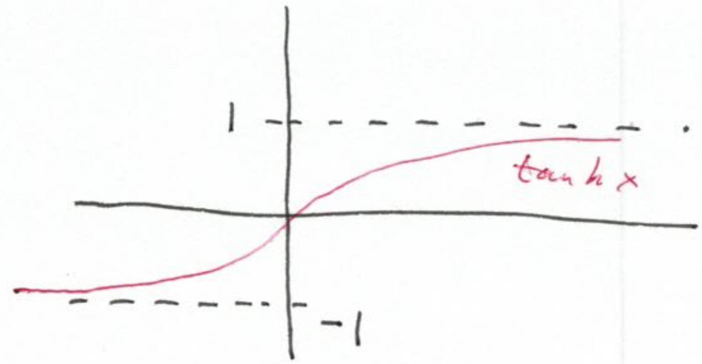
$(\sinh x)' = \cosh x \quad , \quad (\cosh x)' = \sinh x$





Thm
$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$
$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right).$$
$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right).$$

$$\sinh^{-1}(\sinh x) = x$$
$$\sinh(\sinh^{-1} x) = x$$

Check: $\sinh y = \frac{1}{2}(e^y - e^{-y}) = x$ solve y for x to get $y = \sinh^{-1} x$.

$$\Rightarrow e^y - 2x - e^{-y} = 0$$
$$\Rightarrow (e^y)^2 - 2x e^y - 1 = 0 \quad \text{quadratic eq for } e^y.$$

binomial formula: $e^y = \frac{2x}{2} \pm \frac{1}{2}\sqrt{4x^2+4}$

$$= x \pm \sqrt{x^2+1} \quad \geq 0 \Rightarrow \text{take + sign.}$$

$$\Rightarrow e^y = x + \sqrt{x^2+1}$$

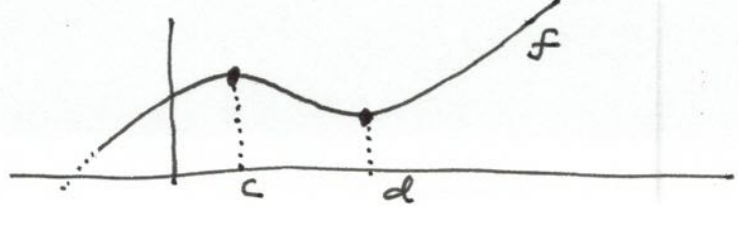
$$\Rightarrow y = \ln\left(x + \sqrt{x^2+1}\right) = \sinh^{-1} x.$$

Applications of differentiation.

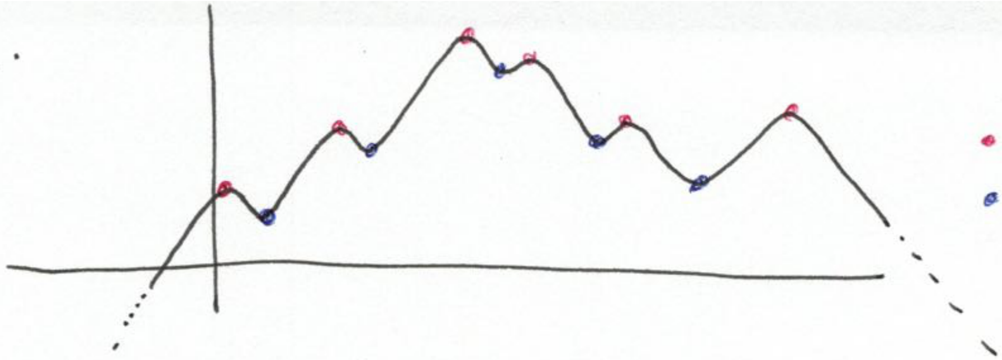
Maximum and minimum values.

Importance: Tool that allows us to optimize things.

Def: f has a local max at c if $f(c) \geq f(x)$ for all x near c.
f has a local min at d if $f(d) \leq f(x)$ for all x near d.



Ex.



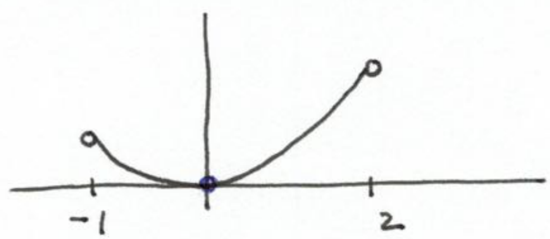
• local max
• local min.

Def A point c in the domain D of f is a global max if $f(c) \geq f(x)$ for all x in D.

A point d in D is a global min if $f(d) \leq f(x)$ for all x in D.

(global min/max = absolute min/max).

Ex $f(x) = x^2$, for $-1 < x < 2$.

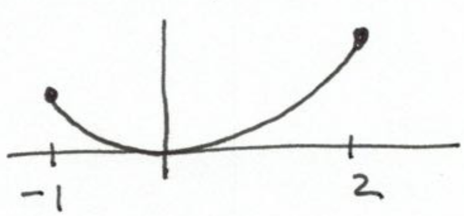


local min at $0 = x$
no local max
global min at 0
no global max.

Ex $f(x) = x^2$, for $-1 \leq x < 2$.

local min at 0
local max at -1
global min at 0.
no global max.

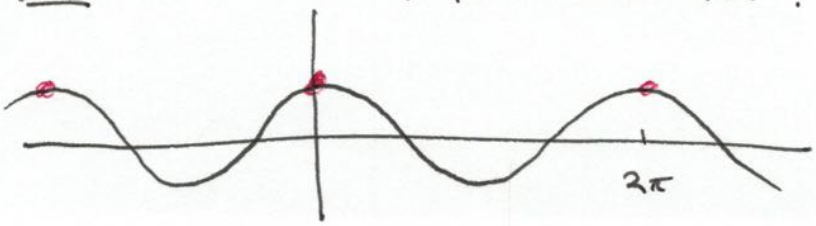
Ex $f(x) = x^2$, for $-1 \leq x \leq 2$



local min at 0

local max at $-1, 2$.
global min at 0
global max at 2.

Ex $f(x) = \cos x$, for $-\infty < x < \infty$.



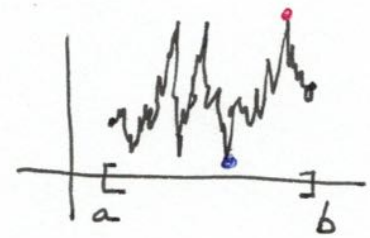
local max at $0, 2\pi, 4\pi, \ldots$
$, -2\pi, -4\pi, \ldots$.

all of them are also global maxima.

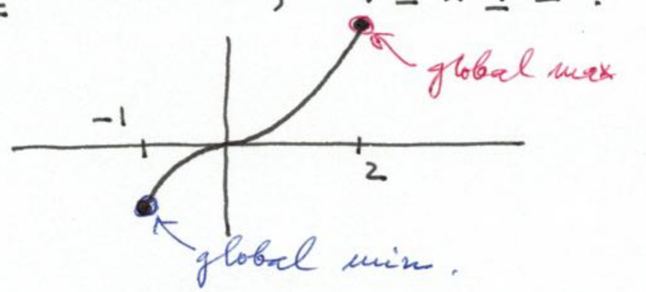
Thm (Extreme value thm).

Assume f is continuous on closed interval $[a, b]$.
Then, f has a global max and a global min in $[a, b]$.

(Proof is difficult, won't do it here).

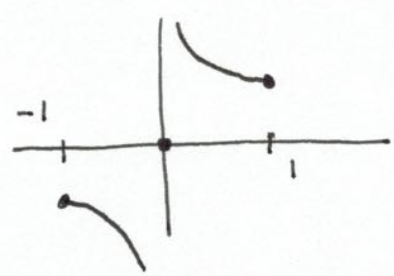


Ex $f(x) = x^3$, $-1 \le x \le 2$.



f contin, interval closed
$\Rightarrow$ extreme value thm applies

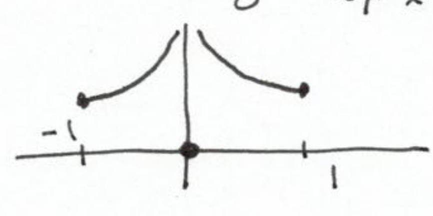
Ex $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ for $-1 \le x \le 1$.



local max at -1
local min at 1.
no global min
no global max
0 is neither local max nor local min.

extreme value thm not applicable: f not continuous.

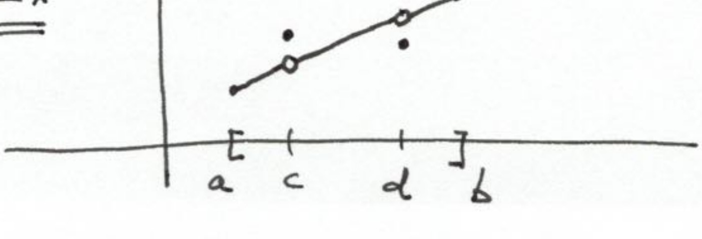
Ex $f(x) = \begin{cases} \frac{1}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ for $-1 \le x \le 1$.



no local max
local min at $-1, 0, 1$
global max none
global min at 0.

extreme value thm not applicable: f not continuous.

Ex



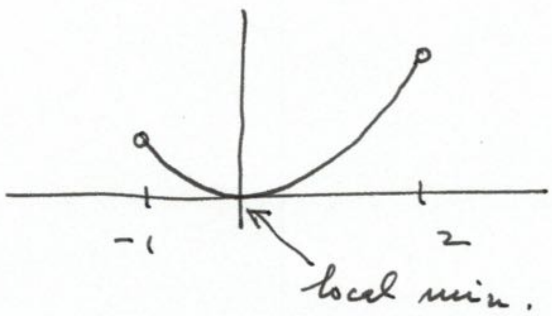
local max b, c
local min a, d.
global max b
global min a.

Thm (Fermat)

Assume f has a local max or local min at c, and that $f'(c)$ exists. ← implies continuous at c.

Then, $f'(c) = 0$

Ex $f(x) = x^2$, $-1 < x < 2$.



$f'(0) = 2 \cdot 0 = 0$