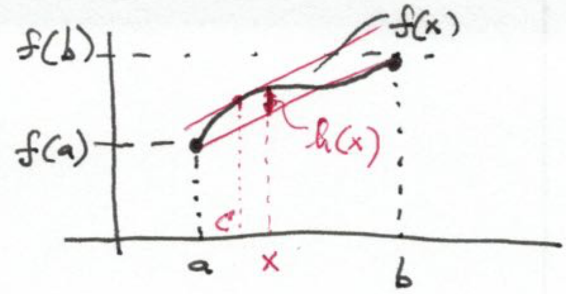
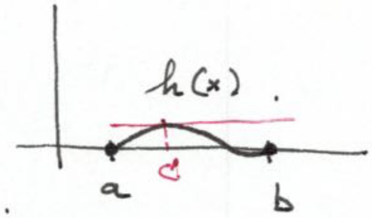
Check (mean value thm).



Connecting line:

$$y = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{slope}} \cdot (x-a) + f(a)$$

$$h(x) = f(x) - \left(\frac{f(b)-f(a)}{b-a} \cdot (x-a) + f(a) \right)$$



Rolle's thm: There is c in (a,b) where $h'(c)=0$.

$$\Rightarrow h'(c) = 0 = f'(c) - \frac{f(b)-f(a)}{b-a} - 0$$

$$\Rightarrow f'(c) = \frac{f(b)-f(a)}{b-a} \quad \text{slope of connecting line.}$$

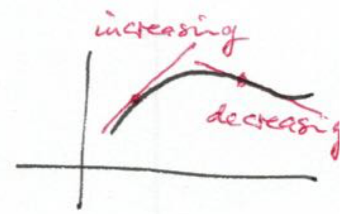
slope of tangent line at $f(c)$

Derivatives & shape of graphs.

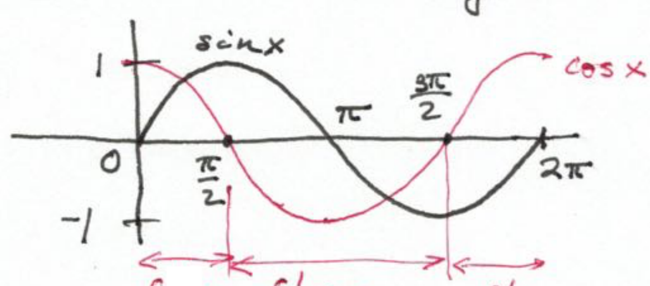
What do we learn from f' about f?

Increasing / decreasing:

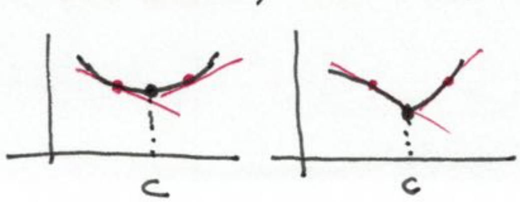
- if $f'(x) > 0$, then f is increasing at x.
- if $f'(x) < 0$, then f is decreasing at x.



Ex $f(x) = \sin x$
$0 \le x \le 2\pi$.
$f'(x) = \cos x$

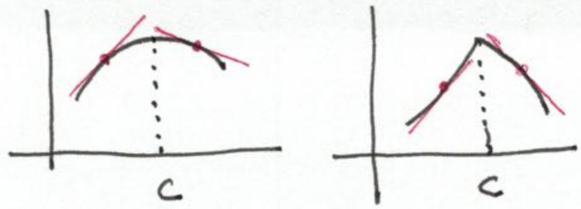


If f is continuous, and c is a critical number ($f'(c)=0$ or $f'(c)$ does not exist).



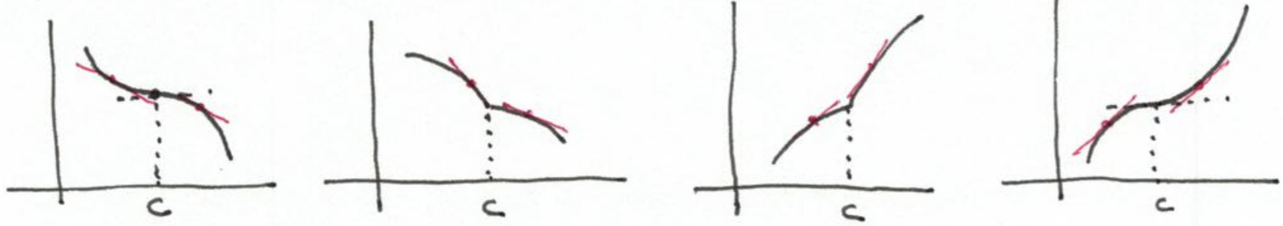
- if f' changes from <0 to >0 at c, then f has a local min at c.

—



if f' changes from >0 to <0 at c, then f has a <u>local max</u> at c.

—



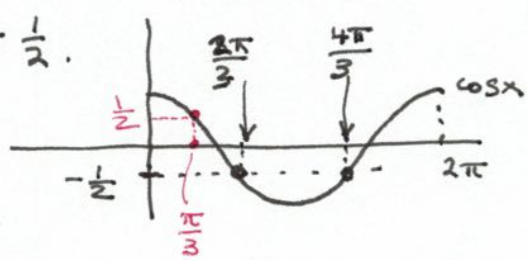
if f' does not change sign at c, then f has <u>no</u> local max or min at c.

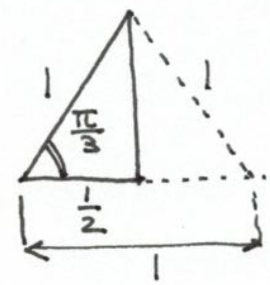
<u>Ex</u> Find local min, max of $f(x) = x + 2\sin x$, $0 \le x \le 2\pi$.

f is continuous on $[0, 2\pi]$, and differentiable in $(0, 2\pi)$

Crit. pts. in $(0, 2\pi)$: $f'(x) = \underbrace{1 + 2\cos x}$ $= 0$ (all crit pts in $(0,2\pi)$ are of this type).

$\cos x = -\frac{1}{2}$.

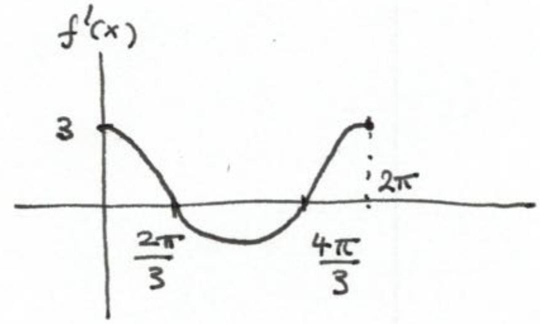




crit. pts. in $(0, 2\pi)$: $\frac{2\pi}{3}$, $\frac{4\pi}{3}$.

Look at signs of f':

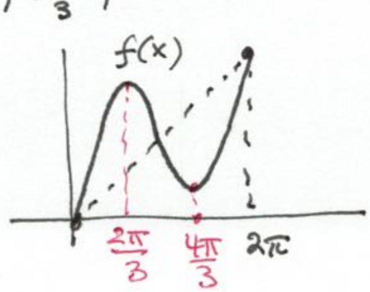
Intervals	$f'(x)$	$f(x)$
$(0, \frac{2\pi}{3})$	+	increasing.
$(\frac{2\pi}{3}, \frac{4\pi}{3})$	—	decreasing.
$(\frac{4\pi}{3}, 2\pi)$	+	increasing.



intervals between crit. pts $0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$.

<u>Crit numbers</u>: $\frac{2\pi}{3}$ local max | 0 local min.

$\frac{4\pi}{3}$ local min | 2π local max.



Second derivative.

f' ~ slope of the tangent line.

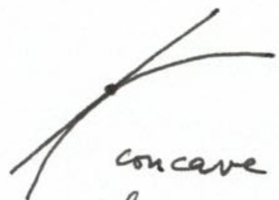
f'' ~ how does the graph of f bend away from the tangent line?

Def:

concave upwards

(~ convex

graph of f is above the tangent



concave downwards.

~ concave).

graph of f is below the tangent.

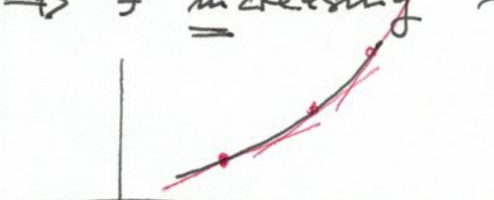
Concavity test: Assume f twice differentiable on (a,b).

Then: 1) if $f''(x) > 0$ for x in (a,b), then f is concave up in (a,b)

2) if $f''(x) < 0$ ——— " ——— concave down —"—

Why? f'' tells you how f' changes.

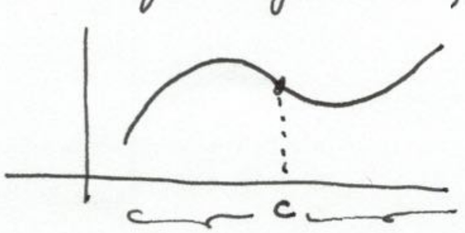
$f'' > 0 \Rightarrow f'$ increasing ~ f gets steeper.



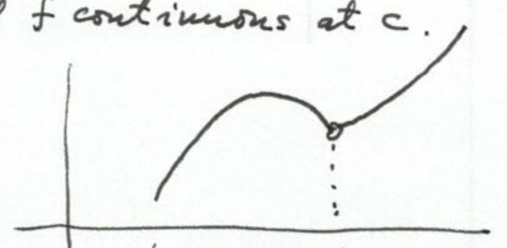
$\Rightarrow$ concave up.

similarly, $f'' < 0 \Rightarrow f'$ decreasing.

Def A point c corresponds to an inflection point if f'' changes sign at c, and f continuous at c.



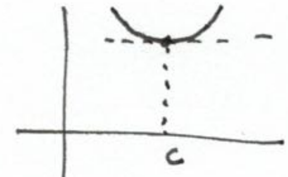
concave down   concave up



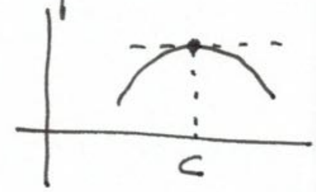
concave down   concave up.

Thm (2nd derivative test) Assume f'' is continuous near c.

1) if $f'(c) = 0$, and $f''(c) > 0$ then f has a local min at c



2) if $f'(c) = 0$, and $f''(c) < 0$ then f has a local max at c.



Remark: If $f'(c)$ does not exist, then $f''(c)$ also doesn't exist, and we can't use the 2nd derivative test. Then, we have to check if f' changes sign at c.