$\underline{\underline{Ex}}$ $\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}} = \lim_{x \to \infty} e^{\frac{1}{x} \ln(e^x + x)}$

$$= e^{\lim_{x \to \infty} \frac{1}{x} \ln(e^x + x)} = e^1 = e$$

if limit exists; exponential is continuous.

$$\lim_{x \to \infty} \frac{\ln(e^x + x)}{x} \underset{\text{de l'H}}{=} \lim_{x \to \infty} \frac{\frac{1}{e^x + x} \cdot (e^x + 1)}{1} = \lim_{x \to \infty} \frac{e^x + 1}{e^x + x}$$

$$\underset{\text{de l'H}}{=} \lim_{x \to \infty} \frac{e^x}{e^x + 1} \underset{\text{de l'H}}{=} \lim_{x \to \infty} \frac{\cancel{e^x}}{\cancel{e^x}} = 1.$$

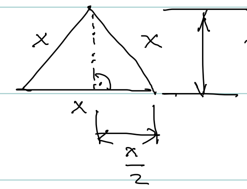
Ex $\displaystyle \lim_{t\to\infty} t \ln\left(1+\frac{3}{t}\right) = \lim_{t\to\infty} \frac{\ln\left(1+\frac{3}{t}\right)}{\frac{1}{t}}$

$$\underset{\text{de l'H}}{=} \lim_{t\to\infty} \frac{\frac{1}{1+\frac{3}{t}}\cdot\left(\frac{-3}{t^2}\right)}{-\frac{1}{t^2}} = \lim_{t\to\infty} \frac{1}{1+\frac{3}{t}}\cdot\left(\frac{+3}{\cancel{t^2}}\right)\cdot\left(+\cancel{t^2}\right)$$

$$= \lim_{t\to\infty} \frac{1}{1+\frac{3}{t}}\cdot 3 = \underline{3}.$$

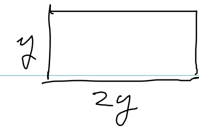
Ex 30 cm wire cut into 2 pieces.

one piece $\longrightarrow$ triangle, equilateral



$$\frac{\sqrt{3}}{2}x = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \sqrt{\frac{3}{4}x^2}$$

other piece $\longrightarrow$ rectangle



Where to cut the wire to have max area enclosed?

Optimization: Area $A = \frac{\sqrt{3}}{4}x^2 + 2y^2$

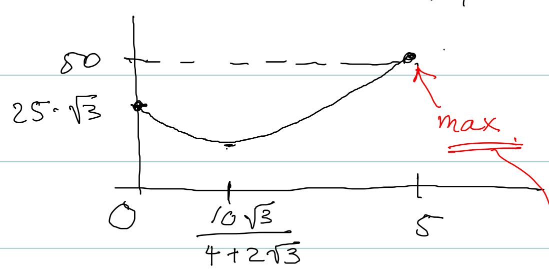
Constraint: $3x + 6y = 30 \Rightarrow x + 2y = 10 \Rightarrow x = 10 - 2y$

Plug into $A = \frac{\sqrt{3}}{4}(10-2y)^2 + 2y^2$. domain of y: $[0,5]$

Derivative $A'(y) = \frac{\sqrt{3}}{4}\cdot 2(10-2y)\cdot(-2) + 4y = -10\sqrt{3} + 2\sqrt{3}y + 4y = 0$

$\Rightarrow -10\sqrt{3} + (4+2\sqrt{3})y = 0 \Rightarrow y = \frac{10\sqrt{3}}{4+2\sqrt{3}}$ in $[0,5]$.

Min/max? $A''\left(\frac{10\sqrt{3}}{4+2\sqrt{3}}\right) = 4 + 2\sqrt{3} > 0$ min!



$A(0) = \frac{\sqrt{3}}{4}\cdot 100 + 2\cdot 0 = 25\cdot\sqrt{3}$.

$A(5) = \frac{\sqrt{3}}{4}\underbrace{(10-2\cdot 5)^2}_{0} + 2\cdot 5^2 = 50$

$y = 5 \Rightarrow$ only rectangle.

Ex Assume x, y satisfy $6y^2 + x^2 = 2 - x^3 \cdot e^{4-4y}$

Determine y' when $x = -2$, $y = 1$.

Check: $6 \cdot 1^2 + (-2)^2 \stackrel{?}{=} 2 - (-2)^3 \cdot e^{4-4\cdot 1}$

$6 + 4 \stackrel{?}{=} 2 - (-8) \cdot e^0$ ✓

Implicit differentiation. $12y \cdot y' + 2x = -3x^2 e^{4-4y} - x^3 e^{4-4y} \cdot (-4y')$

$\Rightarrow$ $12 \cdot 1 \cdot y' + 2(-2) = -3(-2)^2 e^{4-4\cdot 1} - (-2)^3 e^{4-4\cdot 1}(-4y')$

$12y' - 4 = -12 - 32y'$

$\Rightarrow$ $44y' = -8 \Rightarrow y' = \frac{-8}{44} = \underline{\underline{-\frac{2}{11}}}$

Ex Find $f(5.01)$ when $f(x) = 3x\, e^{2x-10}$.

$\Rightarrow$ Linear approximation: $a = 5$.

$$f(5) = 3 \cdot 5 \cdot e^{2 \cdot 5 - 10} = 15.$$

$$f'(x) = 3\, e^{2x-10} + 3x\,(2)\, e^{2x-10}$$

$$f'(5) = 3 + 6 \cdot 5 = 33$$

Linearization $y = f(a) + f'(a)(x-a)$

$\Rightarrow$ $y = 15 + 33(x-5)$. $x = 5.01$.

$\Rightarrow$ $f(5.01) \approx 15 + 33 \cdot 0.01 = 15.33$

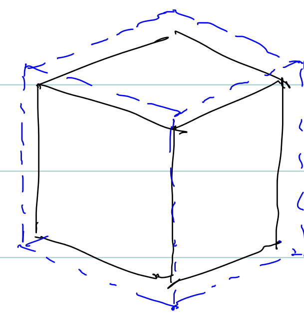
Ex Square box of side lengths a.

By how much does the volume change when all sides increase by Δx when $a = 1\,ft$.

Volume $V = a^3$

Rate of change as a differential

$$\Delta V = 3a^2 \cdot \Delta x$$



$$a = 1\,ft \Rightarrow \Delta V = 3 \cdot \Delta x$$