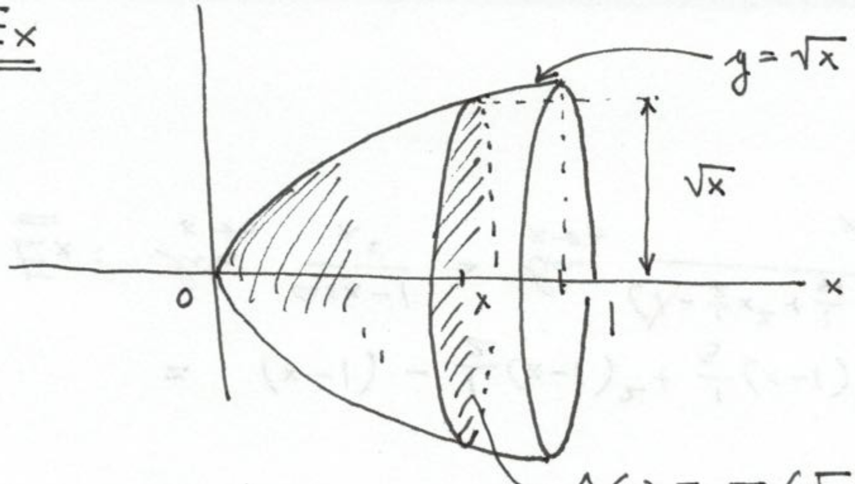
Ex

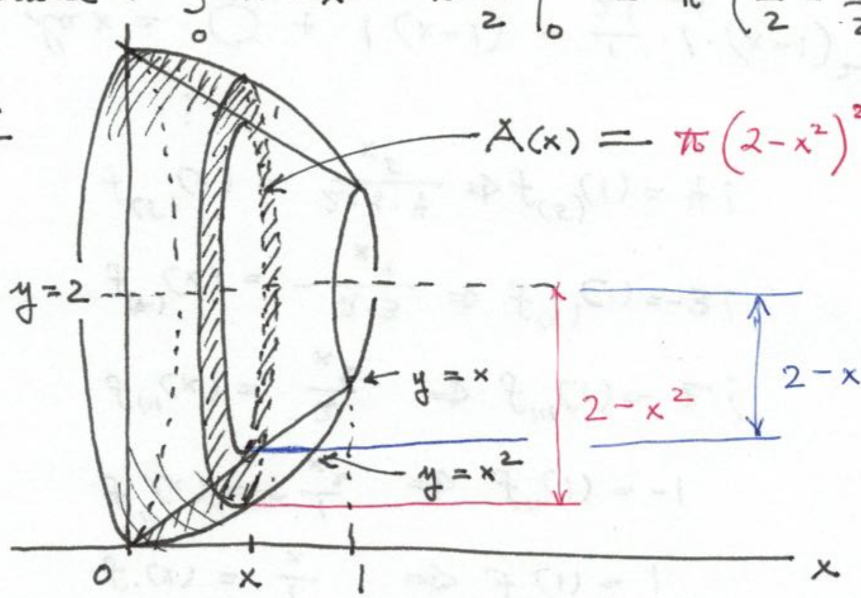


$$A(x) = \pi\left(\sqrt{x}\right)^2 = \pi x.$$

$$\text{Volume} = \int_0^1 \pi x \, dx = \pi \frac{x^2}{2}\Big|_0^1 = \pi\left(\frac{1}{2} - \frac{0}{2}\right) = \frac{\pi}{2}.$$

Ex

$$A(x) = \pi\left(2-x^2\right)^2 - \pi\left(2-x\right)^2$$



$$\text{Volume} = \int_0^1 A(x)\,dx = \pi\int_0^1 \left((2-x^2)^2 - (2-x)^2\right)dx$$

$$= \pi\int_0^1 \left(\cancel{4} - 4x^2 + x^4 - \left(\cancel{4} - 4x + x^2\right)\right)dx.$$

$$= \pi\int_0^1 \left(-5x^2 + x^4 + 4x\right)dx$$

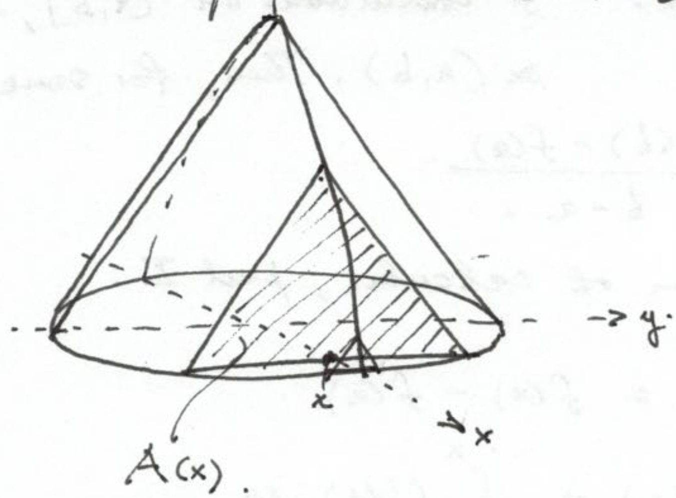
$$= \pi\left(-5\frac{x^3}{3} + \frac{x^5}{5} + 4\frac{x^2}{2}\right)\Big|_0^1$$

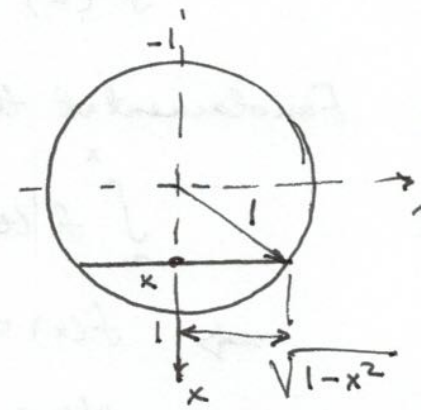
$$= \pi\left(-\frac{5}{3} + \frac{1}{5} + 2 - 0\right).$$

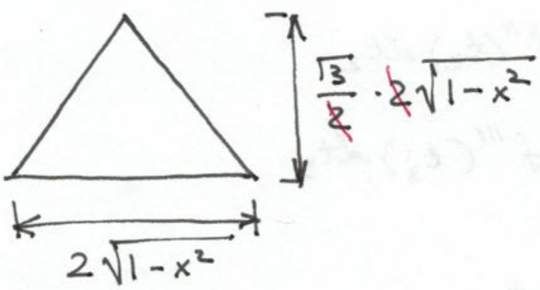
$$= \pi\left(\frac{-25+3+30}{15}\right) = \pi\frac{8}{15}.$$

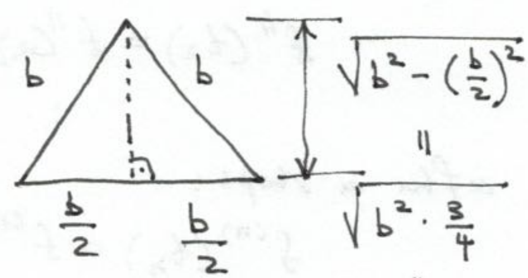
Ex: Solid with a circular base of radius 1.

Cross-sections perpendicular to x-axis are equilateral triangles









$$A(x) = \frac{1}{\cancel{2}} \underbrace{\cancel{2} \cdot \sqrt{1-x^2}}_{\text{base}} \; \underbrace{\sqrt{3}\sqrt{1-x^2}}_{\text{height}}$$

$$= \sqrt{3}\,(1-x^2).$$

$$\text{Volume} = \int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} \sqrt{3}\,(1-x^2)\,dx.$$

$$= \sqrt{3}\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = \sqrt{3}\left(\underbrace{1-\frac{1}{3}}_{\frac{2}{3}} - \Big(\underbrace{-1-\frac{-1}{3}}_{-\frac{2}{3}}\Big)\right).$$

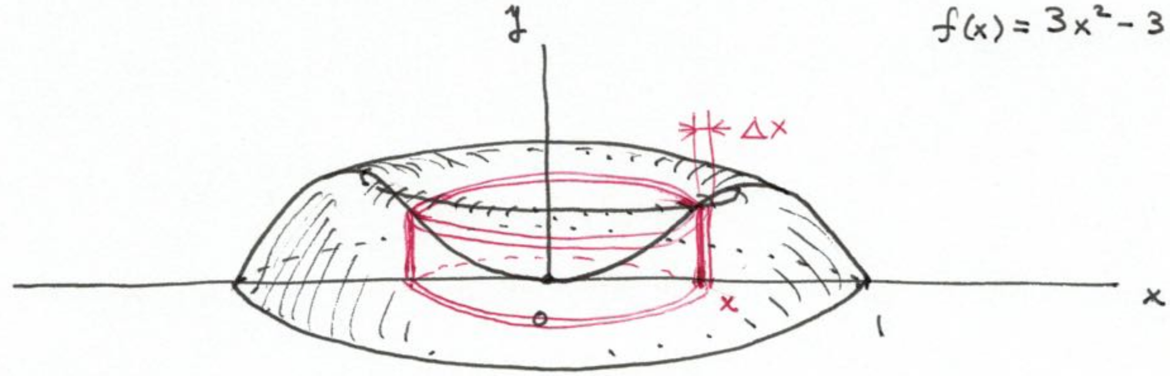
$$= \sqrt{3}\;\frac{4}{3}$$

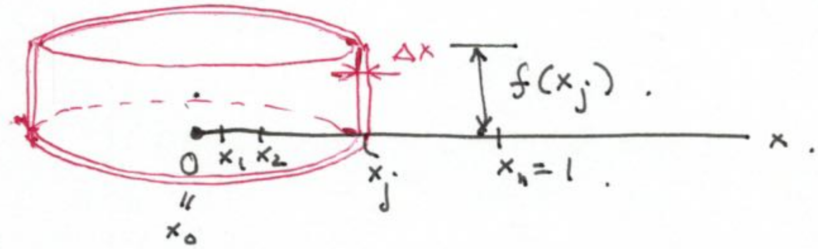
$$\sqrt{b^2-\left(\frac{b}{2}\right)^2} = \sqrt{b^2-\frac{b^2}{4}} = \sqrt{\frac{3b^2}{4}} = \frac{\sqrt{3}}{2}\,b$$

Volumes using cylindrical shells.

$f(x) = 3x^2 - 3x^3$.

Ex





Area $A(x_j) = \underbrace{2\pi x_j}_{\text{circumference}} \cdot \underbrace{f(x_j)}_{\text{height}}$.

Volume $A(x_j) \cdot \Delta x$

Total volume $= \underbrace{A(x_1)\Delta x + A(x_2)\Delta x + \dots + A(x_n)\Delta x}_{\text{Riemann sum}}$.

As $n \to \infty$ and $\Delta x \to 0$,

$\to$ Volume $= \int_0^1 A(x)\,dx = \int_0^1 2\pi x \cdot f(x)\,dx$

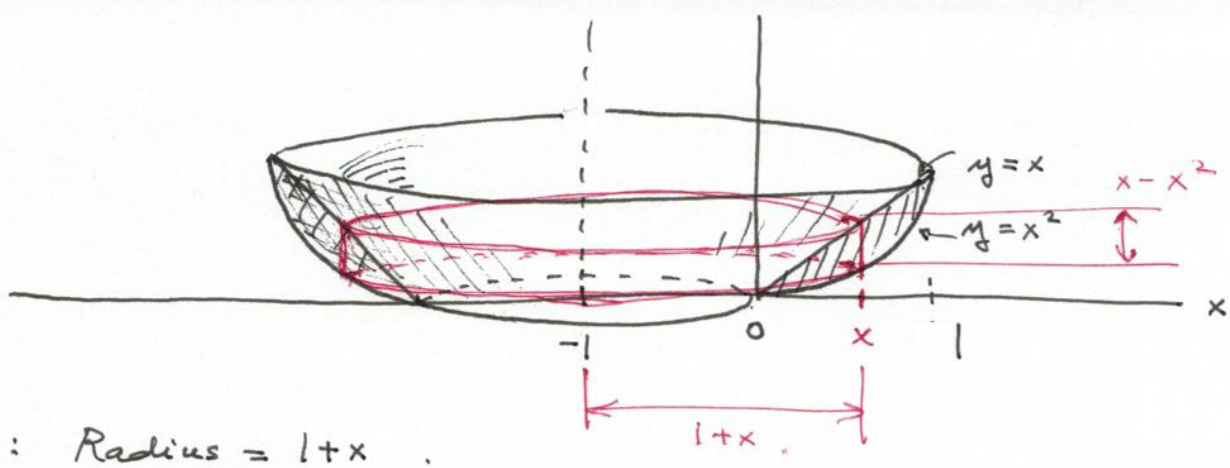
$$\text{Volume} = \int_0^1 2\pi x\,(3x^2 - 3x^3)\,dx.$$

$$= 6\pi \int_0^1 (x^3 - x^4)\,dx.$$

$$= 6\pi \left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Bigg|_0^1 = 6\pi\left(\frac{1}{4} - \frac{1}{5} - 0\right)$$

$$= \frac{6\pi}{20} = \underline{\underline{\frac{3\pi}{10}}}.$$

Ex



Cylinder: Radius $= 1+x$.

Height $= x - x^2$

Area $= \underbrace{2\pi(1+x)}_{\text{circumference}} \cdot \underbrace{(x-x^2)}_{\text{height}}$.

$$\text{Volume} = \int_0^1 A(x)\,dx = \int_0^1 2\pi(1+x)(x-x^2)\,dx.$$

$$= 2\pi \int_0^1 \left(x - \cancel{x^2} + \cancel{x^2} - x^3\right) dx$$

$$= 2\pi\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1 = 2\pi\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \underline{\underline{\frac{\pi}{2}}}.$$