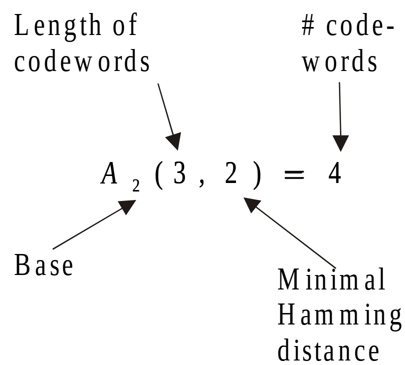
M 375 – Homework 8

4.1.1



Simply by listing all possible codewords, we can proof that there are 4 possible codewords fulfilling the properties given above.

Possible codeword	**Choose**	**Possible codewords**	**Choose**	**Possible codewords**
000		000		000
001		001		
010				
011		011		011
100				
101		101	←	101
110	←	110		110
111				

As one can see, the four possible codewords are {000, 011, 101, 110}. The Hamming distance between each of them is 2.

4.2.1

Given formulae (1) and (3) on page 52. $e = \left\lfloor \frac{d-1}{2} \right\rfloor = \left\lfloor \frac{3-1}{2} \right\rfloor = 1$

$$A_2(10,3) \cdot \sum_{k=0}^{1} \binom{10}{k} (2-1)^k \le 2^{10}$$

$$A_2(10,3) \cdot [1+10] \le 2^{10}$$

$$A_2(10,3) \le 1\frac{1024}{11} \le 93$$

$$A_2(10,3) \cdot \sum_{i=0}^{2} \binom{10}{k} (2-1)^i \geq 2^{10}$$
$$A_2(10,3) \cdot [1+10+45] \geq 2^{10}$$
$$A_2(10,3) \geq \frac{1024}{56} \geq 19$$

Hence, $19 \leq A_2(10,3) \leq 93$.

4.3.2

In order to get all codewords; take all the linear combinations of the rows of the matrix given in 4.3.1.

10111
01101
11000
11010
10101
01111
00010
00000

There are 8 codewords with a Hamming distance of 1.

p. 72, problem 11

In general, an even weighted codeword added to another even weighted codeword gives an even weighted codeword (even + even = even); an odd weighted codeword added to another odd weighted codeword gives an even weighted codeword (odd + odd = even); and an odd weighted codeword added to an even weighted codeword gives an odd weighted codeword (even + odd = odd).

A linear code forms a vectorspace; i.e. any linear combination of codewords yields another codeword.

There are three possible codes:

- All even weighted: According to the observations above, every combination of these codewords yields an even weighted codeword. *All codewords have even weight.*
- All odd weighted: *That's not possible*. At least the 0-codeword, which must be part of every linear code (vectorspace property), is an even weighted codeword.
- Even and odd codewords: *There are half even weighted codewords and half odd weighted codewords*. Proof: If there would exit a linear code with *n* codewords and *n-1* codewords would be even weighted and only 1 codeword would be odd weighted, then it would be impossible to produce all even weighted codewords by adding an even weighted codeword to an odd one that yields an odd one). The vectorspace property wouldn't hold. Every situation with more even weighted codewords than odd weighted codewords would produce more odd weighted codewords. The baseline is, that the even and odd weighted codewords must be distributed evenly (half / half).