1. (5 pts) Evaluate the indefinite integral

$$\int \frac{4}{(1+5x)^3}dx$$

Solution.

Let $u = 1 + 5x$, $du = 5dx$, $dx = \frac{1}{5}du$,

$$\begin{aligned}
\int \frac{4}{(1+5x)^3}dx &= \int \frac{4}{u^3}\frac{1}{5}du \\
&= \frac{4}{5}\int u^{-3}du + C \\
&= \frac{4}{5}\frac{u^{-2}}{(-2)} + C \\
&= -\frac{2}{5}u^{-2} + C \\
&= -\frac{2}{5u^2} + C \\
&= -\frac{2}{5(1+5x)^2} + C
\end{aligned}$$

2. (5 pts) Evaluate the indefinite integral

$$\int_0^{\frac{\pi}{2}} \frac{2\sin x}{3+2\cos x}dx$$

Solution.

Take $u = 3 + 2cosx$, then $du = -2sinxdx$, so $2sinxdx = (-1)du$.
When $x = 0$: $u = 3 + 2 \cdot cos(0) = 5$. When $x = 1$: $u = 3 + 2 \cdot cos(\frac{\pi}{2}) = 3$.
By U-substitution, chaging both the variable to u and the upper, lower limits,

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \frac{2sinx}{3+2cosx}dx &= \int_5^3 \frac{1}{u}(-1)du \\
&= \int_3^5 \frac{1}{u}du \\
&= \ln|u|\Big|_{u=3}^{u=5} \\
&= \ln 5 - \ln 3 \\
&= \ln\left(\frac{5}{3}\right)
\end{aligned}$$

Or: you can find an anti-derivative first, or use several U-subs, for example:

$$
\begin{aligned}
\int \frac{2sinx}{3+2cosx}dx &\overset{u=cosx}{=} \int \frac{2}{3+2u}(-1)du \\
&\overset{v=3+2u}{=} \int \frac{-1}{v}dv \\
&= -\ln|v| \\
&= -\ln|3+2cosx|
\end{aligned}
$$

and $-\ln|3+2cosx|\Big|_{x=0}^{x=1} = (-\ln(3)) - (-\ln 5) = \ln 5 - \ln 3 = \ln\left(\frac{5}{3}\right)$.