1. (5 pts) Find the definite integral

$$I = \int_0^{\frac{\pi}{2}} e^x cosx dx$$

Solution 1. Do integration by parts twice, both with $u = e^x$.

$$\begin{aligned}
I &= \int_0^{\frac{\pi}{2}} e^x cosx dx \\
&\quad (u = e^x, dv = cosxdx \\
&\quad \text{then } du = e^x dx,\ v = sinx) \\
&= (e^x sinx)|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} sinx e^x dx \\
&= e^{\frac{\pi}{2}} \cdot 1 - e^0 \cdot 0 - \int_0^{\frac{\pi}{2}} sinx e^x dx \\
&= e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} sinx e^x dx \\
&\quad (u = e^x, dv = sinxdx \\
&\quad \text{then } du = e^x dx,\ v = -cosx) \\
&\quad (\text{Here use } u = sinx \text{will give} \\
&\quad \text{you the same integral you start with.}) \\
&= e^{\frac{\pi}{2}} - \left((e^x(-cosx))|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-cosx) e^x dx \right) \\
&= e^{\frac{\pi}{2}} - \left((e^{\frac{\pi}{2}} 0 - e^0(-1)) + \int_0^{\frac{\pi}{2}} cosx e^x dx \right) \\
&= e^{\frac{\pi}{2}} - \left(1 + \int_0^{\frac{\pi}{2}} cosx e^x dx \right) \\
&= e^{\frac{\pi}{2}} - 1 - I
\end{aligned}$$

Therefore $2I = e^{\frac{\pi}{2}} - 1$, $I = \frac{e^{\frac{\pi}{2}} - 1}{2}$.

Solution 2. Another way to do integration by parts: with $dv = e^x dx$ twice. We do an 'indefinite integral' version below. The antiderivative you get is the same as in Solution 1.

$$\begin{aligned}
I &= \int e^x cosx dx \\
&\quad (u = cosx, dv = e^x dx \\
&\quad \text{then } du = -sinxdx,\ v = e^x) \\
&= cosx e^x - \int e^x(-sinx) dx
\end{aligned}$$

$$
\begin{aligned}
&= cosxe^x + \int e^x sinxdx \\
&\quad (u = sinx,\ dv = e^x dx \\
&\quad \text{then } du = cosxdx,\ v = e^x) \\
&\quad (\text{Here use } u = e^x \text{will give} \\
&\quad \text{you the same integral you start with.)} \\
&= cosxe^x + sinxe^x - \int e^x cosxdx \\
&= cosxe^x + sinxe^x - I
\end{aligned}
$$

Therefore $2I = cosxe^x + sinxe^x,\ I = \frac{cosxe^x + sinxe^x}{2}\Big|_0^{\frac{\pi}{2}}$.
2. (5 pts) Find the integral

$$I = \int \frac{1 + \sin x}{\cos^2 x} dx$$

Solution.

$$
\begin{aligned}
I &= \int \frac{1 + \sin x}{\cos^2 x} dx \\
&= \int \frac{1}{\cos^2 x} dx + \int \frac{\sin x}{\cos^2 x} dx \\
&= \int \sec^2 x dx + \int \frac{-1}{u^2} du \text{ (set } u = \cos x) \\
&= \tan x + \frac{1}{u} + C \\
&= \tan x + \frac{1}{\cos x} + C \\
&= \tan x + \sec x + C
\end{aligned}
$$